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POLARIZATION OF X-RAYS FROM A TUNGSTEN TARGET

By P. A. Ross

On page 433 of this issue the writer has described a new method of x-ray spectrum analysis especially adapted to dealing with faint sources. This method has here been applied to a preliminary study of the polarization of the continuous radiation from the tungsten target of a standard Coolidge tube.[1]

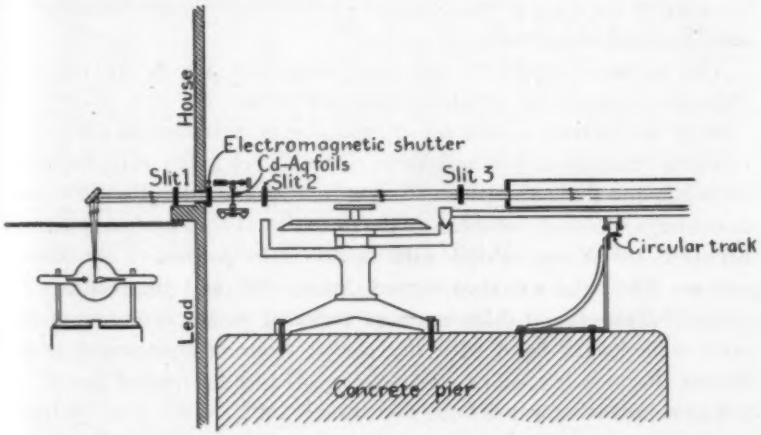


FIG. 1. *General scheme of apparatus for the application of the balanced foil method to the study of polarization.*

Fig. 1 shows the essentials of the set up. An x-ray tube was mounted in a carrier free to rotate about a vertical axis passing through the focal spot of the target and the center of the scattering block. A divided

[1] P. A. Ross, Phys. Rev., *28*, p. 425A; 1926.

scale on the tube carrier provided for sufficiently accurate setting of the cathode stream with reference to the slit system. A graphite block served to analyze the state of polarization of the incident radiation by the familiar method of 90° scattering. Since it was found that the air in front of the slits gave an easily measurable amount of scattered radiation this effect was eliminated by the use of a lead glass plate between the tube and analyzing block. A small hole in the plate permitted only the graphite block to be illuminated by the x-rays and limited the scattering angle to $90° \pm 2°$. An electromagnetic shutter operated by impulses from a standard laboratory clock served to give "exposures" that were integral multiples of seconds to an accuracy of about 1/300 of a second. A selector clock that has been described elsewhere[2] automatically counted the proper number of seconds and made the opening and closing switch contacts.

The electrometer used was a modified form of Compton electrometer built in the department shop by Mr. Burton Stuart. In this work it was used at a sensitivity of about 4000 scale divisions per volt. By a simple potentiometer the sensitivity of the electrometer and the linearity of the scale could be checked at any time, or the electrometer used as a null instrument.

The voltage applied to the x-ray tube was steady dc voltage.[3] Recorded voltages are certainly within 100 volts.

With the cathode stream set at right angles to the line of slits, electrometer readings of the ionization current were taken with balanced cadmium and silver foils alternating in front of the slits. The difference in readings was then recorded as the intensity of the band of radiation between .4850A and .4632A with *electric vector parallel to the cathode stream*. The tube was then turned through 90° and the readings repeated. This second difference was recorded as the intensity of the same wave-length band with the *electric vector at right angles to the cathode stream*.

The polarization was defined as

$$P = \frac{I_1 - I_2}{I_1 + I_2}$$

where P = Polarization.

[2] Webster & Hennings, Phys. Rev., *21*, pp. 301–311; 1923.
[3] D. L. Webster, Proc. Nat. Acad. Sci., *6*, pp. 264–272; 1920.

I_1 = Intensity of radiation with electric vector parallel to cathode stream.

I_2 = Intensity of radiation with electric vector perpendicular to cathode stream.

Values of the polarization and of the intensities I_1 and I_2 for various voltages are given in the accompanying table and also shown graphically in Fig. 2.

		Seconds	Cathode stream perpendicular to scattering plane			Cathode stream parallel to scattering plane			
Kilovolts	Milliamps	Time	Cadmium	Silver	Dif.	Cadmium	Silver	Dif.	Polarization
60.00	1.000	30.00	993	944		843	791		
			995	944		843	793		
			996	945		844	792		
			996	945		842	792		
			995	944		843	793		
			995	944		843	793		
			997	945		844	792		
			994	944		843	792		
			997	945		842	792		
			995	944		843	791		
			995.2	944.2	51.0	843.0	792.1	50.8	.002
55.00	1.000	30.00	734.8	689.0	45.8	626.8	580.8	46.0	
50.00	7.00	6.00	641.6	586.6	55.0	534.8	479.9	54.9	.001
45.00	2.00	30.00	548.2	481.6	66.4	451.4	387.0	64.4	.015
40.00	6.00	20.00	701.4	593.3	108.1	568.8	466.8	102.	.029
35.00	10.00	20.00	676.5	544.4	132.1	536.2	420.6	115.6	.067
32.50	12.00	30.00	868.0	680.4	187.6	662.8	516.8	146.0	.125
30.00	12.00	30.00	582.0	447.1	134.9	404.0	318.0	86.	.28
29.00	12.00	30.00	465.4	355.4	110.0	303.4	255.5	47.9	.39
28.00	12.00	30.00	349.6	269.4	80.2	221.8	204.8	17.	.65
27.50	12.00	60.00	576.6	536.6	40.0	377.2	375.4	1.8	.92
27.30	12.00	60.00	558.3	528.4	29.9	356.6	355.0	1.6	.90
27.00	12.00	60.00	526.9	511.8	15.1	326.8	327.0	−.2	
26.50	12.00	60.00	488.3	483.2	5.1	295.4	295.2	.2	
26.00	12.00	60.00	457.5	455.6	1.9	269.8	269.9	−.2	

In this table complete readings of electrometer deflections are given only for 60 kv. For all other voltages only average deflections are recorded here.

It will be noticed that the polarization, as defined above, is approximately zero for exciting voltages over about twice the "quantum voltage" of the wave-length band being studied. As the applied voltage approaches the quantum voltage (i.e., as the short-wave limit of the white radiation approaches the wave-length of the isolated band) the polarization increases rapidly with every indication that there would be complete polarization at the high frequency limit. It might be well to point out that the pure quantum theory of Bubb[4] as to the origin

[4] F. W. Bubb, Phys. Rev., *24*, pp. 177–189; 1924.

of the general radiation would require complete polarization at the short wave limit. This would also be required by the classical theory of Bergen Davis.[5] On the other hand, the theory of Kramers[6] would seem to prohibit more than a partial polarization, even at the high frequency limit.

In Fig. 2 bracket *A* indicates the position and width of the isolated band in terms of quanta voltages. Bracket *B* indicates the "modified" position of the same band as computed from the Compton effect

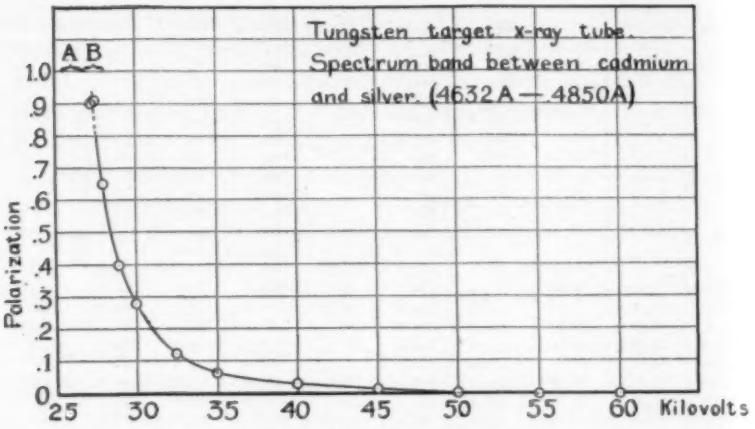


FIG. 2. *Polarization of the general x-radiation.*

formula $\Delta\lambda = .0484 \sin^2\theta/2$. It will be seen that the polarization curve is heading approximately for the modified band *B*. However, data recorded in the tables at 27 kv, 26.5 kv, and 26 kv indicate the presence of some unmodified radiation, as might be expected from a graphite scattering block.

Fig. 3 shows isochromats plotted from data given in the tables. Here again brackets *A* and *B* indicate the unmodified and modified positions of the isolated spectrum band.

By certain slight modifications and extensions, the method of balanced foils promises to be of much value in investigations of the Compton effect. Even in the simple form used here it enables this effect to be demonstrated with ease and without the use of special x-ray

[5] Bergen Davis, Phys. Rev., *9*, pp. 64–77; 1917.
[6] H. A. Kramers, Phil. Mag., *46*, pp. 836–871; 1923.

tubes, high energy in-put, ultra-sensitive electrometers or interminable exposures such as were essential before.

It is hoped that this work may be extended to many other spectral bands, to higher voltages, to an investigation of the state of polarization

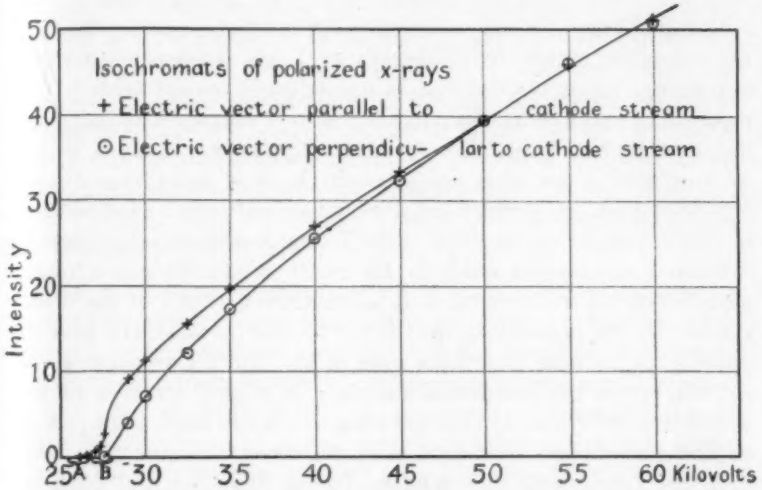


FIG. 3. *Isochromats of the general x-radiation showing the relation of form to the direction of the electric vector.*

of radiation from other target materials, to polarization of characteristic lines isolated between absorption discontinuities, and particularly to the state of polarization of radiation from a thin target.

DEPARTMENT OF PHYSICS,
STANFORD UNIVERSITY,
CALIFORNIA.

CONJUGATE SURFACES OF A SPHERICAL REFRACTING SURFACE

By James P. C. Southall

1. *Introduction.*—In a presidential address delivered before the Optical Society in London, 10th February, 1927, Mr. Thomas Smith read a noteworthy paper entitled "Some uncultivated optical fields." The report of this address as now published in the *Transactions of the Optical Society* (*28*, No. 5, pp. 225–284: 1926–27)was not received in this country until a few days ago after the lapse of more than a year; otherwise, I am confident attention would have already been directed to it. Incidentally, the same issue of the journal mentioned above, besides containing also another article by Mr. Smith, is adorned with a frontispiece which will be recognized as an excellent portrait of the distinguished former president of the Optical Society by those who have the privilege of knowing him. Like most of Mr. Smith's writings on geometrical optics, the presidential address is an original contribution in a field which unfortunately does not seem to have so much attraction for natural philosophers nowadays as it did about a century ago when Professor (afterwards Sir) William Rowan Hamilton published his series of brilliant papers on the "Theory of systems of rays" in *The Transactions of the Royal Irish Academy*. In his opening sentence Mr. Smith explains why he chose this subject for his address on this occasion, because (as he says) "within the whole domain of optics I suppose there is no region in which more of the present members of the Optical Society are vitally interested than that which we know as geometrical optics." It is certainly true that under the leadership of Mr. Smith and his coworkers England has regained much of the prestige in applied optics which she seemed to have lost in the two or three decades preceding the great war.

Mr. Smith makes a strong plea for returning to Hamilton's neglected work and making a more practical use of the famous "characteristic function," as Hamilton called it, and of a similar function developed by H. Bruns in 1895 called the *eikonal*. It is all very interesting, I daresay important, and I, for one, wish Mr. Smith the best of luck in his quest. If anyone succeeds in finding a "northwest passage" through the labyrinth in which the optical designer finds himself enmeshed at present, I have great hopes that it will be Mr. Smith himself, whether he

follows Hamilton or some other of the earlier explorers. The greater part of the presidential address is taken up with this analytical investigation. It is not my purpose to take part in that discussion until I have had time to study the whole subject more fully.

In the earlier portion of his paper where Mr. Smith beings forward some new graphical methods in geometrical optics he is in lighter vein. I have always believed that more could be accomplished by geometrical constructions than is commonly supposed, and apparently Mr. Smith is of the same opinion. His investigation of the so-called secondary imagery of a spherical refracting surface is certainly original and suggestive. In a certain sense the following paper may be considered as a commentary on pages 229–237 of the presidential address. It contains also some new points of view.

2. *Single spherical refracting surface: The two kinds of imagery.*—A symmetrical optical instrument, such as an ordinary telescope, microscope or photographic lens, is one in which any meridian section made by a plane containing the axis of symmetry or optical axis is precisely like any other meridian section. It is generally composed of a series of glass lenses, whose surfaces are portions of spheres with their centers of curvature all lying on the optical axis. The apertures of the bundles of rays traversing the system are regulated by a circular diaphragm or stop which is suitably located in a transversal plane perpendicular to the axis with its center on this line. The chief ray of a bundle is then defined as that one of the rays which in traversing the medium in which the aperture-stop is situated passes through its center.

Since the entire instrument forms a centered system of spherical refracting surfaces, it is of fundamental importance to investigate the laws of a single spherical refracting surface, as has been done by many previous writers and again in an original fashion by Mr. Smith in his Presidential Address. Incidentally, it will be necessary to define certain symbols and remind the reader of a few general laws with which he is doubtless already familiar.

The angles of incidence and refraction denoted by α and α' are acute angles which are connected with each other by the law of refraction, namely,

$$n' \sin \alpha' = n \sin \alpha,$$

where n and n′ denotes the indices of refraction of the two media that are separated by the spherical refracting surface. The optical axis is a

fixed straight line which goes through the center C of the surface and meets the effective portion of it at its vertex designated by A. The step from A to C will be denoted by r, that is, the radius of the surface is $r = AC$.

The following construction of the path of a ray refracted at a spherical surface was given by Huygens in his *Dioptrica*[1] and also by Thomas Young in his lectures on Natural Philosophy[2]:

With radii equal to $(n'/n)r$ and $(n/n')r$ describe two spheres, σ and σ', respectively, concentric with the given spherical refracting surface. If the straight line RB (Fig. 1) represents the path of a ray incident

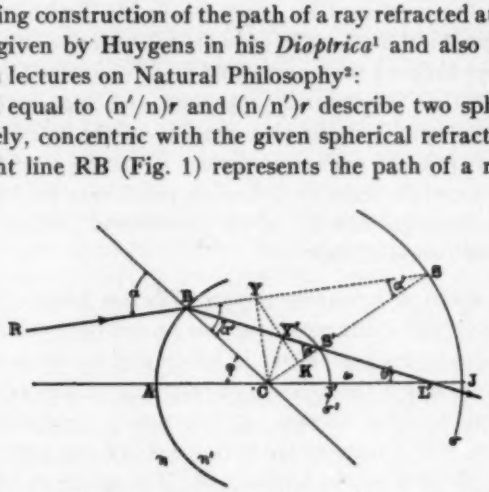


FIG. 1. *Young's construction of ray refracted at a spherical surface. Straight lines* BS, BS' *show path of ray before and after refraction. Points* C *and* K *are centers of perspective of two ranges of conjugate points on this ray.*

on the refracting surface at B, and if this straight line meets the surface σ in S, the path of the refracted ray BL' will be along the straight line BS', where S' designates the point where the straight line CS crosses the surface σ'. Of course, a straight line meets the surface of a sphere in two points, and in making this construction care must be taken to select S as that one of the two points of intersection for which the segments BS and BS' of the incident and refracted rays will both lie on the same side of the incidence-normal BC, in accordance with the law of refraction.

The diameter of the stop is supposed here to be comparatively small so that the apertures of the bundles of effective rays are all narrow. It is well known that there are always two principal sections of a

[1] Huygens' *Œuvres complètes, 13*, p. 63.—See also Southall's *Mirrors, Prisms and Lenses*, revised ed., p. 617, New York, 1923.

[2] *A course of lectures on Natural Philosophy and the Mechanical Arts*, by Thomas Young, M.D., London, 1807, in two volumes. See *2*, Art. 425, p. 73.

narrow bundle of rays lying in mutually perpendicular planes which both contain the chief ray. The so-called meridian section is the section made by the plane of incidence of the chief ray, whereas the other principal section called the sagittal section is in the plane perpendicular to the plane of incidence. Thus the bundle contains two distinguished pencils of rays, a pencil of meridian rays lying in the plane of incidence or primary principal plane (which is usually the plane of the diagram) and a pencil of sagittal rays lying in the secondary principal plane. Since the chief refracted ray lies in the plane of incidence, the primary principal plane of the bundle of refracted rays coincides with that of the bundle of incident rays; whereas the secondary principal plane of the bundle of refracted rays will generally not be the same as that of the bundle of incident rays, but these planes will intersect each other in a line perpendicular to the plane of incidence of the chief ray at the point of incidence.

On the supposition that the incident rays all pass through a single point constituting a monocentric bundle, the peculiar distinction that is characteristic of both the pencil of meridian rays and the pencil of sagittal rays is that these pencils, and generally these only, will be monocentric after refraction. Thus if before refraction the rays all emanated, let us say, from a point-source O, then after refraction the meridian rays will all intersect the chief refracted ray in a point O′, and the sagittal rays will likewise all intersect it in a point O″ which is generally not the same as O′; whereas all the other rays not contained in either principal section will each pass close to both O′ and O″ without actually intersecting the chief refracted ray at all. In such a case the bundle of refracted rays is not monocentric but astigmatic. In a certain sense we may say that O′ and O″ are both images of the point O, one being the primary image and the other the secondary image, although it should not be inferred from these ordinal numbers that one of the images is superior to the other. As a matter of fact, the so-called secondary image may be considered as the better of the two, inasmuch as the covergence of the sagittal rays at O″ is more perfect than that of the meridian rays at O′. Accordingly, we have here a two-to-one correspondence (instead of a one-to-one correspondence) in the sense, namely, that if P, Q, R, S, etc., is a range of points lying on the chief ray of a narrow bundle of incident rays, there will be two corresponding ranges P′, Q′, R′, S′, etc., and P″, Q″, R″, S″, etc., both lying on the chief refracted ray. Now there is a lucky circumstance, so to speak,

about each of these two corresponding ranges of points, and that is, both of them have a center of perspective. For instance, considering one kind of imagery at a time, suppose that E (Fig. 2) is the position of the center of perspective of two ranges of points lying on two straight lines intersecting in the point B (or B′). Then if P, Q, R, S, etc., is one of these ranges, the corresponding perspective range P′, Q′, R′, S′, etc., can be found immediately, because the straight lines PP′, QQ′, RR′, SS′, etc., must all pass through the center of perspective at E. Evidently, the point B (or B′) where the straight lines PQ and P′Q′ intersect is a double or self-corresponding point of both ranges.

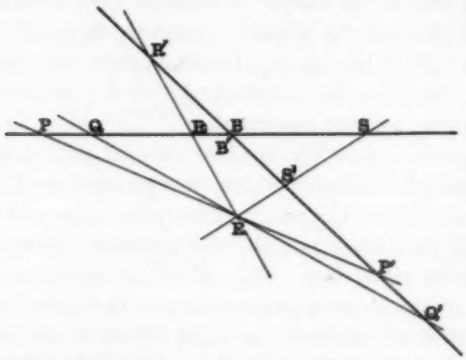


FIG. 2. P, Q, R, S *and* P′, Q′, R′, S′ *are two perspective ranges of points with respect to point* E *as center of perspective.*

In the case of the secondary or sagittal imagery the center of perspective coincides with the center of curvature C of the spherical surface, as is evident by mere considerations of symmetry, and as has been known ever since the discovery of the law of refraction. It was Thomas Young who seems to have been the first to recognize the existence of the other center of perspective K (Fig. 1) for the primary imagery by means of the meridian rays.[3]

It is easy to find the position of the point K for a given incident ray, since all we have to do is to draw the central perpendicular to this ray, namely, CY (Fig. 1), and then draw YK perpendicular to CS at K. (Or we can also draw CY′ perpendicular to the refracted ray and Y′K perpendicular to CS′. Evidently Y, Y′ and S, S′ are two pairs of corresponding primary points.) Since the straight line joining S, S′ passes through both centers of perspective, this pair of points has the

[3] Thomas Young, On the mechanism of the human eye, Phil. Trans., *92*, p. 23; 1801.

distinction of being a corresponding pair of separate points for both kinds of imagery; and hence if the incident rays are all directed towards S, the refracted rays will all pass through S′. The interval between the two centers of perspective is

$$CK = \frac{n}{n'} r \sin^2 \alpha.$$

Accordingly, the position of K depends on the angle of incidence of the chief ray. Thus there is an extremely significant difference between the two centers of perspective, for while the center of perspective C for the secondary imagery is an absolutely fixed point independent of any particular ray, the position of the other center of perspective K for the primary imagery is variable and different for one ray from what it is for another ray. This fact constitutes a fundamental distinction between the two kinds of imagery that is not always kept in mind by writers on geometrical optics. It enables us to see also why the secondary imagery, as above stated, is more perfect than the primary imagery, and leads us to anticipate that the formulation of the laws of the primary imagery is likely to be not so simple as that of the secondary imagery.

3. *The Magnification-ratio* m.—Having constructed the path of the chief ray, we proceed now to investigate the so-called magnification at a point P in one of the two principal sections. In the plane of this section imagine a tiny little line PV drawn through P perpendicular to the chief incident ray. The image of this line will be found to lie in the corresponding principal section of the bundle of refracted rays, and will be also a short line P′V′ perpendicular to the chief refracted ray at the point P′ conjugate to P. The magnification-ratio in this principal section is

$$m = \frac{P'V'}{PV}.$$

The value of m will depend on the position of the point P, and it may be positive or negative. It will depend also on the chief ray and the angle of incidence. Moreover, it is not the same for one principal section as for the other.

4. *The image-equations.*—In order to have a precise idea of these relations, let us proceed now to formulate the image-equations, with-

out explaining how they are derived, although that would not be difficult.[4] Let P, P′ designate the positions of a pair of corresponding points on the chief incident and refracted rays, respectively, and let B designate the position of the point of incidence. Put $q = \mathrm{BP}$, $q' = \mathrm{BP}'$, where the distances denoted by q and q' are to be reckoned positive or negative according as these steps measured from the point of incidence are in the same direction as the light travels along the ray or in the opposite direction, respectively. Moreover, following Gullstrand and other modern writers, let us put

$$Q = \frac{\mathrm{n}}{q}, \quad Q' = \frac{\mathrm{n}'}{q'},$$

where Q, Q' will be expressed therefore in dioptries provided q, q' are given in meters. If m denotes the magnification-ratio, the image-equations may be written as follows:

$$\beta^2 Q' = Q + \beta D, \quad \beta \mathrm{m} Q' = Q,$$

where β and D are two functions which have a constant value for a given spherical refracting surface and given chief ray, although these functions are not the same for both principal sections. The magnitude denoted by β is the value of m when P and P′ coincide with each other at the point of incidence. In the sagittal section $\beta = 1$ for all values of α; whereas in the meridian section β is a function of α, namely, $\beta = \cos\alpha'/\cos\alpha$. The other magnitude, denoted by D, is the measure of the refracting power of the system along the chief ray in question, and is therefore a function of α. Thus for the two principal sections the functions denoted here by β and D are found to be as follows:

$$\textit{Sagittal Section}: \quad D = \frac{\mathrm{n}'\cos\alpha' - \mathrm{n}\cos\alpha}{r}, \quad \beta = 1.$$

$$\textit{Meridian Section}: \quad D = \frac{\mathrm{n}'\cos\alpha' - \mathrm{n}\cos\alpha}{r\cos\alpha\cos\alpha'}, \quad \beta = \frac{\cos\alpha'}{\cos\alpha}.$$

5. *Imagery in the sagittal section. Pair of conjugate spheres for which the value of the magnification-ratio is constant.*—Accordingly, the image-

[4] See, for example, A. Gullstrand, Appendix I of Vol. I of English Translation of Helmholtz's *Physiological Optics* (published by the Optical Society of America, 1924), pp. 272–277. —Also, Southall's *Principles and Methods of Geometrical Optics*, 2nd ed. (New York, 1913), Appendix to Chapter XI; and Southall's *Mirrors, Prisms and Lenses*, revised ed., New York, 1923, pp. 529–532.

equations in the sagittal section of a narrow bundle of rays refracted at a spherical surface are:

$$Q' = Q + D, \quad \mathrm{m}Q' = Q,$$

where $D = (\mathrm{n}'\cos\alpha' - \mathrm{n}\cos\alpha)/r$; and hence the magnification-ratio, $\mathrm{m} = Q/(Q+D)$, is a function of Q and α. Using the law of refraction ($\mathrm{n}'\sin\alpha' = \mathrm{n}\sin\alpha$) to eliminate α', we shall obtain the following quadratic equation in Q:

$$\left(\frac{1-\mathrm{m}}{\mathrm{m}}\right)^2 Q^2 + \frac{1-\mathrm{m}}{\mathrm{m}} \cdot \frac{2\mathrm{n}}{r} \cdot Q\cos\alpha - \frac{\mathrm{n}'^2 - \mathrm{n}^2}{r^2} = 0.$$

If a and k are functions of the magnification-ratio and defined as follows:

$$a = \frac{1-\mathrm{m}}{\mathrm{m}} \cdot \frac{\mathrm{n}^2}{\mathrm{n}'^2 - \mathrm{n}^2} \cdot r, \qquad k = \frac{\mathrm{n}'}{\mathrm{n}} a,$$

this quadratic equation may be transformed into

$$q^2 + a^2 - 2aq\cos\alpha = k^2,$$

where $q = Q/\mathrm{n}$, as was stated above. Evidently therefore k represents the side of a triangle whose other two sides a and q include an angle equal to α.

Similarly, by eliminating Q and α from the image-equations and putting $q' = \mathrm{n}'/Q'$, we obtain:

$$q'^2 + a'^2 - 2a'q' \cdot \cos\alpha' = k'^2,$$

where a' and k' are functions of m defined as follows:

$$a' = (1-\mathrm{m})\frac{\mathrm{n}'^2}{\mathrm{n}'^2 - \mathrm{n}^2} r = \frac{\mathrm{n}'^2}{\mathrm{n}^2}\mathrm{m}a, \quad k' = \frac{\mathrm{n}}{\mathrm{n}'} a' = \mathrm{m}k.$$

Thus a', q' and k' represent the sides of another triangle in which the angle α' is equal to the angle included between the sides denoted by a' and q'.

In order to interpret these results in terms of the actual geometrical relations, let us consider Fig. 3 (or Fig. 4) in which the incident and refracted chief rays intersect each other at the point B on the spherical refracting surface ($\alpha = \angle\mathrm{CBS}$, $\alpha' = \angle\mathrm{CBS}'$). If the point P is taken anywhere on the incident ray, the conjugate point P′ for the so-called secondary imagery will be at the point where the straight line joining P with the center C intersects the refracted ray. If straight lines PZ,

P′Z′ are drawn through P,P′ parallel to CS (or CS′) meeting the incidence-normal in Z,Z′, respectively, evidently the triangles BZP, BZ′P′ thus formed will be the triangles above mentioned whose sides are $BP=q$, $BA=a$, $ZP=k$ and $BP'=q'$, $BZ'=a'$, $Z'P'=k'$.

Suppose now that the point B remains fixed while the direction of the chief ray is varied, then the incidence-normal continues the same, that is, the central angle $BCA=\phi=$ constant, while the angle of incidence (α) changes. Under these circumstances, for a constant value

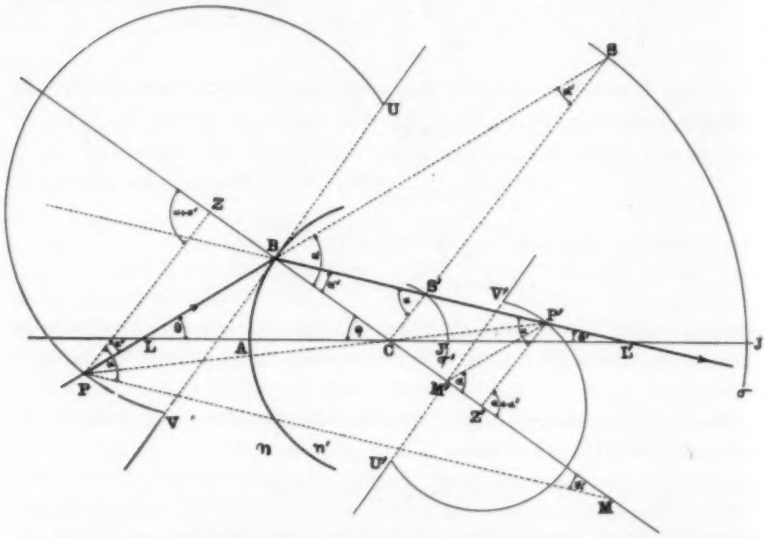


FIG. 3. *Conjugate spheres for a fixed point of incidence and for a given value of the magnification-ratio* m. *The centers of these spheres are at* Z *and* Z′ *and their radii are* ZP *and* Z′P′. *The effective zones are* UPV *and* U′P′V′.

of the magnification-ratio m the magnitudes denoted by a, k and a', k' will be constant also, and the loci of the pairs of conjugate points P and P′ will be the surfaces of the spheres described around Z and Z′ as centers with radii ZP and Z′P′, respectively. These radii are parallel to each other; $\angle PZB=\angle P'Z'B'=\pi-(\alpha+\alpha')$.

When $\alpha=0$, the chief ray is incident normally at B, and the points P and P′ are situated on the incidence-normal at the points where it intersects the pair of conjugate spheres. If in this case the point B coincided with the vertex A of the spherical refracting surface ($\phi=0$),

the bundle of rays would be a bundle of *paraxial rays*, and the transversal planes tangent to the spheres at P and P′ would be a pair of conjugate planes for paraxial imagery.

In general, therefore, for the so-called secondary imagery with respect to a fixed point of incidence B on a spherical refracting surface, the point P corresponding to a definite value of the magnification-ratio m

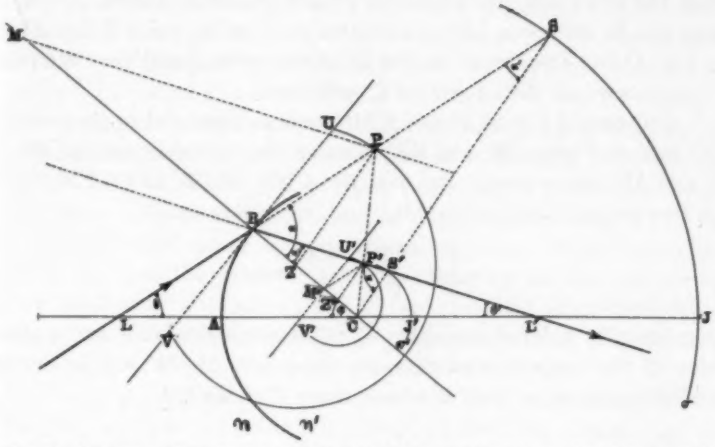


FIG. 4. *Conjugate spheres for a fixed point of incidence and for a given value of the magnification-ratio* m. *The centers of these spheres are at* Z *and* Z′ *and their radii are* ZP *and* Z′P′. *The effective zones are* UPV *and* U′P′V′.

will lie on the surface of a sphere, the center of which is on the incidence-normal at the following distance from the center of the spherical refracting surface:

$$CZ = \frac{n^2 - mn'^2}{m(n'^2 - n^2)} r;$$

and, similarly, the conjugate point P′ will lie on the surface of a sphere with its center also on the incidence-normal at the following distance from the center of the spherical refracting surface:

$$CZ' = m \cdot CZ = \frac{n^2 - mn'^2}{n'^2 - n^2} r.$$

So far as I am aware, Mr. Smith is the first writer on geometrical optics who has recognized the existence of these conjugate spheres in the case of the secondary imagery of a spherical refracting surface.

It appears to me to be an unusually useful and important contribution to our knowledge of the subject. There is no similar simple correspondence in the case of the primary imagery, partly owing to the fact that the center of perspective K for the meridian rays is not a fixed point as is the center of perspective C for the sagittal rays. Mr. Smith uses a purely geometrical method in deriving his results, different from the analytical process which I have employed above. Thus, for example, he shows that for a given value of m the point Z (see Fig. 3 or Fig. 4) is a fixed point on the incidence-normal and that the point P is at a constant distance from Z, as follows:

If the straight lines PM and P′M′ are drawn parallel to the incident and refracted rays BS and BS′ meeting the incidence-normal BC in M and M′, respectively, the triangles CBP, CM′P′ and CPM, CP′B are two pairs of similar triangles, and, consequently,

$$\frac{CP'}{CP}=\frac{CM'}{CB}=\frac{CB}{CM}=m.$$

Therefore, for a definite position of the incidence-point B and a given value of the magnification-ratio m, the points M, M′ will be on the incidence-normal at fixed distances from C, namely,

$$CM=-\frac{r}{m},\quad CM'=-m\cdot r,$$

where $r=BC$ denotes the radius of the spherical refracting surface. Evidently, since $CM\cdot CM'=r^2$, M and M′ are harmonic conjugates with respect to B and the opposite end of the diameter of the sphere. Moreover, it is obvious that $\angle MPZ=\angle ZBP=\alpha$, $\angle ZMP=\angle BPZ=\alpha'$, and therefore, since the triangles ZPM and ZBP are similar,

$$\frac{ZP}{MZ}=\frac{BZ}{ZP}=\frac{\sin\alpha'}{\sin\alpha}=\frac{n}{n'}.$$

Hence $BZ=(n/n')\cdot ZP=(n^2/n'^2)\cdot MZ$; and, since $MZ=MC+CB+BZ=(1-m)/r+BZ$, we find, after putting $BZ=a$ and $ZP=k$:

$$a=\frac{1-m}{m}\cdot\frac{n^2}{n'^2-n^2}\cdot r,\quad k=\frac{n'}{n}a,$$

which are exactly the same results as were previously obtained. Analogously, from the similar triangles Z′P′M′ and Z′BP′, we may derive the corresponding expressions for $a'=BZ'$ and $k'=Z'P'$.

The diagrams (Figs. 3 and 4) have been drawn for the case when the second medium is optically denser than the first ($n' > n$); and in this case the limiting values of the angle of incidence are $\alpha = \pm(\pi/2)$, $\alpha' = \pm A$, where A denotes the critical angle of the two media ($\sin A = n:n'$). Now when $\alpha = \pm(\pi/2)$, the incident ray will be tangent to the refracting sphere at the point B, and the point P will be at V or U where the circle described around Z as center with radius $k = ZP$ meets the tangent drawn to the arc of the circle AB at B. Moreover, since $\angle Z'M'P' = \alpha$, when $\alpha = \pm(\pi/2)$, $M'P'$ will be perpendicular to BC at M' and the point P' will be at V' or U'. Hence the effective portions of the conjugate spheres described around Z and Z' are limited by the planes perpendicular to the incidence-normal at B and M', respectively.

As has been noted, the radii of a pair of conjugate spheres are always in the same ratio to each other as the magnification-ratio ($k'/k = m$). In each case the radius is proportional to $\mu/[(\mu+1)(\mu-1)]$, where $\mu = n'/n$ denotes the relative index of refraction of the two media. Assuming that $n' > n$, that is $\mu > 1$, we see therefore that, other things being equal, the dimensions of the conjugate spheres will diminish with increase of the relative index of refraction.

6. *The conjugate spheres in special cases.*—The incidence-normal BC remaining fixed, as the point P is shifted on the chief incident ray, the value of the magnification-ratio $m = CP'/CP$ will vary, and, consequently, the positions of the centers Z, Z' of the conjugate spheres and of the points M, M' on the incidence-normal and the magnitudes of the radii $k = ZP$, $k' = Z'P'$ will vary likewise. In particular, when one of the conjugate points P, P' is infinitely remote, the magnification-ratio has the value 0 or ∞, and then one of the conjugate spheres is the infinitely distant plane of space, whereas the radius of the other so-called focal point sphere has the same magnitude in either case; moreover, one of the harmonic conjugate points, M, M' will coincide with the center C of the spherical refracting surface, and the other will be the infinitely distant point of the incidence-normal. Thus, for example, if $m = 0$, the incident sagittal rays will be a pencil of parallel rays, which after refraction will intersect in the focal point I', as the point P' will be designated in this case (Fig. 5); and we have here

$$a' = \frac{n^2}{n'^2 - n^2} r, \quad k' = \frac{nn'}{n'^2 - n^2} r, \quad (m = 0),$$

the point M' being at C. The locus of I' is the portion of the sphere described around Z' as center with radius $Z'I'$, limited by U' and V',

as shown in the diagram. It is interesting to observe that this focal point sphere was found seventeen years ago by E. v. Högh.[5] On the

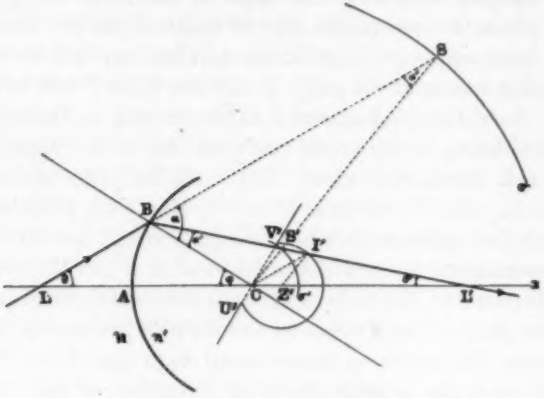


FIG. 5. *Focal point sphere for magnification-ratio* m=0.

other hand, if m = ∞, the narrow pencil of refracted rays will be parallel, and the corresponding incident rays will meet now in the other focal point I (as the point P is designated in this special case), lying on the

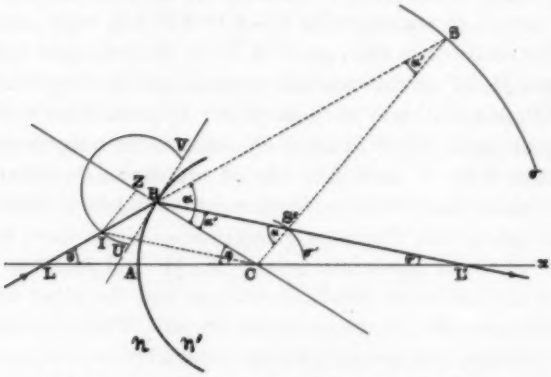


FIG. 6. *Focal point sphere for magnification-ratio* m = ∞.

portion of the sphere shown in Fig. 6 described around Z as center and limited at U and V by the tangent-plane to the spherical refracting

[5] E. v. Högh, Dioptrische Untersuchungen. *C. P. Goerz Festschrift* (Berlin-Friedenau, 1911), pp. 171–175.—See also Southall's *Principles and Methods of Geometrical Optics*, second ed., Appendix to chap. XI, page 366a, New York, 1913.

surface at the incidence-point B. The point M coincides now with C, and

$$a=-\frac{n'^2}{n'^2-n^2}r, \quad k=-\frac{nn'}{n'^2-n^2}r, \quad (m=\infty).$$

When the magnification-ratio has the value unity ($m=+1$), then $a=a'=k=k'=0$; in this case the points P, P′ coincide with each other at the incidence-point B, and the conjugate spheres both collapse into a single point at their common center B.

Lastly, when $m=-1$ (inverted image with unit magnification) the two points M, M′ coincide with each other at the end of the diameter

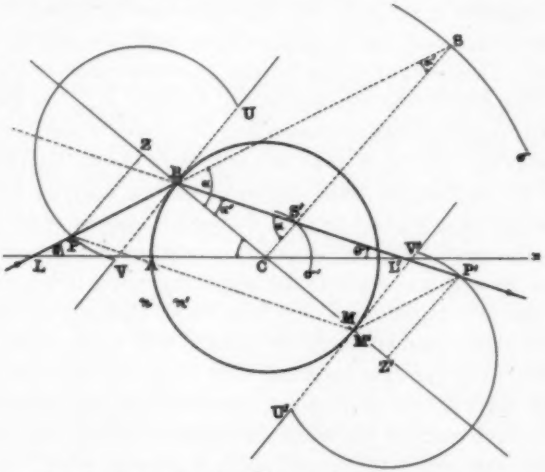


FIG. 7. *Conjugate spheres for magnification-ratio* $m=-1$.

of the spherical refracting surface opposite the incidence-point B, as shown in Fig. 7; and now we have the following relations:

$$CM=CM'=r, \; CZ'=ZC=\frac{n'^2+n^2}{n'^2-n^2}r, \quad k'=-k=\frac{2nn'}{n'^2-n^2}r, \; (m=-1).$$

Here also the dimensions of the two conjugate spheres are equal; their centers being on opposite sides of C and at equal distances from it. The radius of each sphere here is twice that of either of the two focal point spheres.

Thus the positions on the incidence-normal of the centers Z, Z′ of a pair of conjugate spheres will depend on the value of the magnifica-

tion-ratio m. For instance, the point Z may coincide with the center C of the refracting sphere, and then we must have $a = r$ and $k = n'r/n$; accordingly, in this case, for all positions of the incidence-point B, the point P will be identical with the point designated by S ($q = BP = BS = s$), and $m = n^2/n'^2$. Likewise, when $a = r$, we find that $a' = r$, $k' = nr/n'$, and therefore $BZ' = BC$, $Z'P' = CS'$. Accordingly, the pair of conjugate spheres corresponding to the above value of m are the concentric spheres σ and σ' described around C as center with radii $n'r/n$ and nr/n', respectively.

7. *Influence of the stop.*—In the preceding investigation of the secondary imagery of a spherical refracting surface the system of pairs of conjugate spheres was found by confining our attention to a single point of incidence and treating the central angle $\phi = \angle BCA$ therefore as constant. Consequently, the normal to the surface at this point B was a fired straight line containing the centers, Z, Z′ of each of the pairs of conjugate spheres corresponding to a given value of the magnification-ratio m. The fact that the apertures of the bundles of rays concerned in the imagery were said to be narrow may have seemed to imply that the rays all had to pass through a stop of small diameter; otherwise, the existence of the stop was tacitly ignored. It is true, casual mention of the stop was made in the beginning where it was stated that the chief ray of a bundle was that ray which crossed the optical axis at the center of the stop either before refraction or afterwards. The stop may be erected in the first medium, that is, it may be a *front stop*, as it is called, directly controlling the apertures of the bundles of rays before they have reached the refracting surface; but it may just as well be a so-called *rear stop* in the second medium through which the effective rays have to pass after refraction.

In general, the point where a ray crosses the axis (whether "really" or "virtually") before and after refraction will be designated by L and L′, respectively. If this ray is the chief ray of the bundle, the center of the stop must be on the axis at one of these points, but it cannot be at L unless the point L lies in front of the vertex A of the spherical refracting surface, and likewise it cannot be at L′ unless the point L′ lies beyond the vertex. The acute angle made by the ray with the optical axis is the measure of its slope, this angle being denoted by θ before refraction and by θ' afterwards; that is, $\theta = \angle ALB$, $\theta' = \angle AL'B$. There is a simple relation between these angles and the angles of incidence and refraction which is expressed as follows:

$$\alpha' - \theta' = \alpha - \theta = \phi.$$

Each of these five angles is comprised between the limits $+\pi/2$ and $-\pi/2$.

In the following discussion the point L is supposed to be a fixed point on the optical axis through which all the incident chief rays must pass; and hence if $c = \mathrm{CL}$ denotes the abscissa of the point L with respect to the center of the surface as origin, c is to be treated as a constant. Obviously,

$$c \sin\theta = -r \sin\alpha.$$

Now this equation can be employed to eliminate the angle α from the quadratic equation in q which was found above, namely:

$$q^2 - 2aq\cos\alpha + a^2 - k^2 = 0;$$

and if then, after putting $k = \mathrm{n}'a/\mathrm{n}$, the roots of the quadratic are obtained, the following is the value of q that gives the distance of the point P from the point of incidence B, where P designates the position of the point for which the magnification-ratio has the given value:

$$q = \frac{a}{r}\left(\sqrt{r^2 - c^2\sin^2\theta} + \frac{1}{\mathrm{n}}\sqrt{\mathrm{n}'^2r^2 - \mathrm{n}^2c^2\sin^2\theta}\right).$$

The equation connecting q and θ is one of the fourth degree and represents a surface that has four points of intersection with the straight line determined by the path of the chief ray; but the only one of these points that has any interest for us is the point P above mentioned which is located by the above expression for $q = \mathrm{BP}$. For instance, the locus of points P for which the value of the magnification-ratio is infinite is given by the equation

$$q = -\frac{\mathrm{n}^2}{\mathrm{n}'^2 - \mathrm{n}^2}\left(\sqrt{r^2 - c^2\sin^2\theta} + \sqrt{\mathrm{n}'^2r^2/\mathrm{n}^2 - c^2\sin^2\theta}\right), \quad (\mathrm{m} = \infty);$$

and hence for $\theta = 0$, we obtain in this case $q = \mathrm{AF} = -\mathrm{n}r/(\mathrm{n}' - \mathrm{n})$, where F designates the position on the axis of the first focal point of the spherical refracting surface. When $\theta = 0$, the magnitude c disappears from the formula along with θ, the point P lies on the axis, and the bundles of rays are now paraxial rays; and if here we put $q = \mathrm{AP} = u$, the general equation reduces to $u = a(\mathrm{n}' + \mathrm{n})/\mathrm{n} = (1 - \mathrm{m})f/\mathrm{m}$, where $f = \mathrm{FA} = \mathrm{n}r/(\mathrm{n}' - \mathrm{n})$ denotes the first focal length of the spherical refracting surface. This expression for u in terms of m is one of the forms

of the familiar abscissa-relation for paraxial rays. Another simple special case that is also of practical interest occurs when the stop is placed at the center of the spherical surface ($c=0$), so that the chief rays all meet the surface normally; and then, for a given value of m, we find that $q=a(n'+n)/r$ will be a constant for all values of the slope-angle θ, and hence the locus of P will be a sphere concentric with the refracting sphere, its radius being $PC=r-q$. For example, in this latter case ($c=0$), if the magnification-ratio is unity ($m=1$), then $a=0$ and $PC=r$; that is, the locus of P will be the surface of the refracting sphere itself.

A perfectly analogous expression for $q'=BP'$ can be derived in the same way for the locus of the point P′ conjugate to P.

All these results can be likewise obtained geometrically by a method which leads to a simple graphical construction of the conjugate surfaces corresponding to a given value of the magnification-ratio m and for a given position of the stop. The diagram (Fig. 8) has been drawn only for the incident ray whose path is supposed to lie along the straight

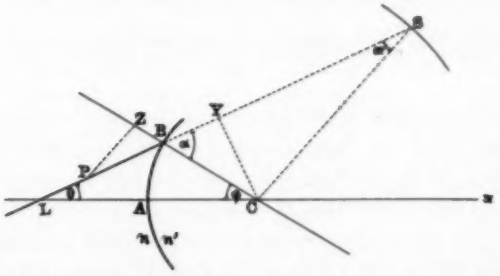


FIG. 8. *Chief incident ray crosses optical axis at* L *and meets spherical refracting surface at* B *and concentric surface* σ *at* S. PZ *is parallel to* CS, *and* CY *is perpendicular to* BS.

line LB which crosses the optical axis at L and meets the refracting surface at B. The point designated by S is the point of intersection of this straight line with the sphere described around S as center with radius $n'r/n$, as used in Young's construction of the refracted ray. From any point P on the incident ray draw the straight line PZ parallel to the straight line CS to meet the incidence-normal at Z, thereby forming the pair of similar triangles BPZ and BSC. Accordingly,

$$\frac{BP}{BS}=\frac{BZ}{BC};$$

and hence, on substituting here the symbols $q=\mathrm{BP}$, $a=\mathrm{BZ}$, $s=\mathrm{BS}$, and $r=\mathrm{BC}$, the following equation will be derived immediately:

$$q=\frac{a}{r}s.$$

Now if Y designates the foot of the central perpendicular on the incident ray, then $\mathrm{CY}=-c\sin\theta$, where $c=\mathrm{CL}$, $\theta=\angle\mathrm{ALB}$; and evidently

$$\mathrm{BY}=\sqrt{r^2-c^2\sin^2\theta},\quad \mathrm{YS}=\sqrt{n'^2r^2/n^2-c^2\sin^2\theta}.$$

Consequently, since $\mathrm{BS}=\mathrm{BY}+\mathrm{YS}$,

$$s=\sqrt{r^2-c^2\sin^2\theta}+\sqrt{n'^2r^2/n^2-c^2\sin^2\theta};$$

which leads therefore to the same expression for q as was found analytically by solving the quadratic equation for q.

For a given value of the magnification-ratio m the magnitudes $a=\mathrm{BZ}$ and $k=\mathrm{ZP}$ are both constant; and hence the position of the point P on any incident ray may be constructed immediately by first laying off on the incidence-normal the step $a=\mathrm{BZ}$, and then with Z as center and radius $k=\mathrm{ZP}$ describing an arc cutting LB in P. Accordingly, we can locate as many such points P as we like for which the value of m will be the same. In exactly similar fashion we can find also the conjugate points P′ on the corresponding refracted rays. This graphical construction appears to be an exceedingly useful practical method of investigating the secondary imagery of a spherical refracting surface, although I have not yet had an opportunity of testing it in actual lens-designing.

I am much indebted to my friend and pupil, Mr. B. Steuer, for drawing the diagrams used to illustrate this paper.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY, NEW YORK CITY.
MARCH 15, 1928.

THERMOLUMINESCENCE EXCITED BY X-RAYS

FURTHER EXPERIMENTS UPON SYNTHETICALLY PREPARED MATERIALS

By Frances G. Wick and Mabel K. Slattery

ABSTRACT

Many materials originally inactive are made thermoluminescent by exposure to x-rays. Calcium sulphate containing a small amount of manganese in solid solution shows this effect strongly and has been previously studied by the authors. The present report gives a continuation of this work upon sulphates of cadmium, calcium, sodium and zinc containing a small amount of manganese.

Exposure to x-rays produces two kinds of change in the material. One of these results in a thermoluminescence which comes up quickly upon heating and lasts for a very short time while the other is shown by a luminescence which comes up more slowly and lasts longer. If the material is allowed to stand at room temperature after exposure the first effect disappears in a few hours while the second may last for months. Both of the effects may be preserved indefinitely by keeping the specimen in liquid air after exposure.

The decay of thermoluminescence was studied using different temperatures of observation, different temperatures of exposure and different voltages on the x-ray tube. Exposure to ultraviolet light of a definite frequency was found to suppress the effect produced by exposure to x-rays. The results obtained are compared with effects of exposure to cathode rays as described by Wiedemann and Schmidt.

The ability of cathode rays to make many materials thermoluminescent has long been known. The effect of exposure to these rays in many cases is to give to the substance a faint phosphorescence which becomes a brilliant thermoluminescence when the specimen is heated. Wiedemann and Schmidt[1] give a list of substances, some pure and some containing a trace of impurity in solid solution, in which thermoluminescence can be produced in this way. For the solid solutions in particular they made a study of the factors which influence the color and brightness of the luminescence.

Somewhat later it was discovered that x-rays were also efficacious in the production of thermoluminescence and a study was undertaken by the authors[2,3] to determine the nature of the luminescence thus imparted. Natural crystals of fluorite and calcite were observed before and after exposure to x-rays and a synthetic material, calcium sulphate containing manganese, the brightest of the samples mentioned by Wiedemann and Schmidt, was made the subject of many observations.

[1] Wiedemann and Schmidt, Ann. der Physik und Chemie, *56*, p. 201; 1896.
[2] Wick, J.O.S.A. & R.S.I., *14*, p. 33; 1927.
[3] Wick and Slattery, J.O.S.A. & R.S.I., *14*, p. 125; 1927.

The variations in the intensity and rate of decay of the luminescence produced by changing the concentration of manganese, the length of exposure to x-rays and the temperature of exposure and of observation were studied.

The work here reported was undertaken to find more about the nature of the effects due to x-rays and to compare them with those observed for cathode rays by Wiedemann and Schmidt.[1] It was hoped that such a comparison might be suggestive of the nature of the change produced by the radiations. The sulphates of cadmium, sodium and zinc with a slight amount of manganese are made thermoluminescent by exposure to x-rays and these were made the subject of this study.

The calcium, sodium and zinc compounds were carefully prepared by adding manganese chloride solution with a small amount of sulphuric acid to the sulphate of the metal, the proportions being such as to make one molecule of manganese to 250 molecules of the calcium sulphate. The mixture, after thorough stirring, was evaporated to dryness and heated until all of the sulphuric acid fumes had disappeared. The cadmium sulphate preparation used contained manganese sulphate in the ratio of one molecule of manganese to 900 of cadmium. This material was some which had been previously used in the laboratory and the method of preparation was not known to the authors.

The samples after being powdered in a mortar and sifted through bolting cloth of 100 mesh to the inch were exposed for 1 hour to x-rays from a molybdenum target tube running on 50 K.V. and 3 milliamperes at a distance of 11 cm from the tube. Immediately after exposure the material was put into long glass tubes immersed in liquid air and kept at this low temperature until the observations could be made. Upon removal from the liquid air the specimens were heated upon a copper plate fastened to a small electric stove, the temperature of the plate being measured by a thermo-junction. Photometric observations of the maximum brightness of the thermoluminescence of the specimen at different temperatures were made and of the rate of decay of this luminescence. Decay curves were plotted in the usual way with the reciprocal square root of intensity as a function of the time after the specimen had reached its maximum brightness. The same methods of exposure and observation were employed as those used in the previous work.[2]

The specimens were kept in liquid air to prevent any loss of energy between the times of exposure and observation. The materials were

all phosphorescent at room temperature after exposure to x-rays and some of the energy absorbed from the x-rays was given off during the necessary time which elapsed between exposure and photometric observation if the material was at room temperature. Since the specimens showed no phosphorescence at liquid air temperature it was assumed that no energy was emitted at this low temperature. In order to investigate this point measurements were made of the maximum brightness of thermoluminescence of all the materials immediately after exposure and after they had stood in liquid air for varying lengths of time. It was found that for all of the specimens except cadmium sulphate the brightness of the thermoluminescence at a given temperature was the same immediately after exposure as it was after the specimen had been cooled, and that with the specimens kept in liquid air, no change took place over a period of several days.

The difference in intensity observed in specimens of cadmium sulphate which had been cooled to liquid air temperature after exposure and that which was observed without cooling was probably not due to the cooling but to the fact that the specimens which were not cooled emitted considerable energy between the time when the exposure ended and the time when the observation could be made in another room. The short time necessary for the manipulation of the experiment seemed to make a difference in the case of cadmium sulphate but not in the case of the other specimens.

The most conspicuous fact brought out in this work was that in every case exposure to x-rays results in a thermoluminescence which comes up quickly upon heating the specimen and lasts for a very short time followed by another luminescence which comes up much less quickly and persists much longer. In some specimens the two emissions are of the same color, in others of different colors. They have been observed[2] previously in the case of natural crystals of fluorite and calcite and ascribed to the presence of more than one impurity in solid solution. If the materials are allowed to stand at room temperature after exposure the first of these effects lasts for only a few hours while the second may last for months or even for years. Both of the effects are preserved by keeping the specimens in liquid air after exposure. Cadmium sulphate shows these two processes especially well. A sample after exposure to x-rays was separated into two parts, one left at room temperature and the other kept in liquid air for an hour. They were then placed side by side on the same piece of metal and heated to a temperature of about 100°C. The one kept in liquid air brightened

then diminished in intensity, then increased in brightness again. Fig. 1, B is a rough sketch of this. The other specimen gradually increased in brightness without showing the two maxima as is shown in Fig. 1, A. When the temperature of observation is considerably higher the effect is similar to that shown in Fig. 1, A, and the whole process is too swift to enable the two maxima to be distinguished.

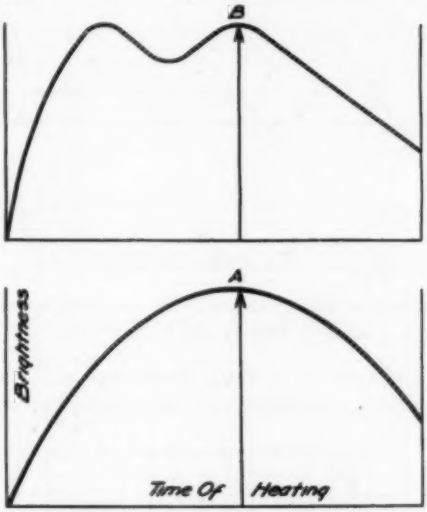


FIG. 1. *Sketch showing character of thermoluminescence maxima.*
A—*Specimen heated after standing at room temperature an hour or more.*
B—*Specimen heated immediately after exposure.*

Zinc sulphate plus manganese showed two maxima, the second slightly brighter than the first. Decay curves for two different temperatures of observation are shown in Fig. 2. Curve A for the lower temperature shows decidedly two processes. In curve *B* the time plotted is that from the second maximum since the first flashed up too quickly at this higher temperature for observation.

Decay curves at two different temperatures for calcium sulphate plus manganese are shown in Fig. 3. Curve *B* taken at 110°C shows an inflection indicating two processes. Only one maximum is visible. This is higher and the rate of decay is more rapid than it is at the lower temperature. At the higher temperature shown in curve *A* the second process has changed from the persistent to the vanishing type.[4]

[4] Nichols and Howes, Nat. Acad. Sci., Proc., *4*, pp. 305; 312.

Results of a similar kind were obtained upon fluorite at a considerably higher temperature range. In every case increasing the temperature

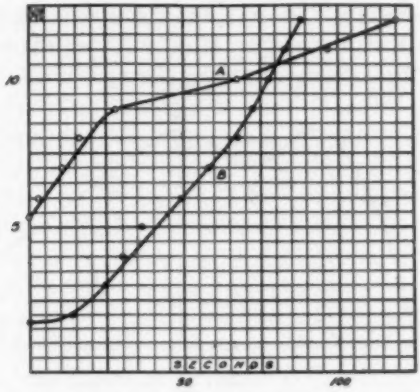


FIG. 2. *Zinc sulphate plus manganese. Decay curves at different temperatures of observation.* A—200°C; B—265°C.

of observation produces a brighter maximum and speeds the decay. The calcium sulphate plus manganese, on standing at room temperature

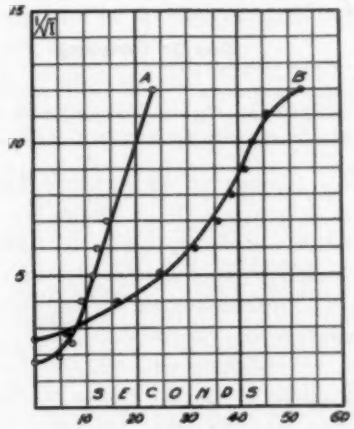


FIG. 3. *Calcium sulphate plus manganese. Decay curves at different temperatures of observation.* A—200°C; B—110°C.

for 17 hours after exposure dropped to 0.17 of its original brightness when the temperature of observation was 200°C.

The thermoluminescence of sodium sulphate plus manganese after exposure to x-rays decays too rapidly to permit of decay curves. It gives two maxima which can be seen easily, the first less bright than the second and more yellow in color. The second has a decidedly orange cast. Lowering the temperature of the material to that of liquid air did not change the brightness of its thermoluminescence upon subsequent heating.

Cadmium sulphate plus manganese showed decidedly two processes in the decay curves. Curve *A*, Fig. 4, shows the decay of a specimen measured as soon as possible after exposure (within about 10 minutes). Curve *B* shows the decay of the same specimen put into liquid air immediately after exposure (within 30 seconds) and observed after

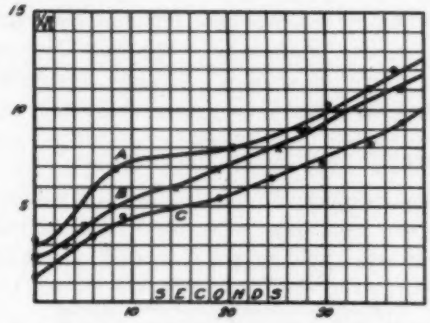


FIG. 4. *Cadmium sulphate plus manganese. Decay curves.*
A—*Temperature of observation 90°C. Specimen not put into liquid air.*
B—*Temperature of observation 90°C. Specimen put into liquid air.*
C—*Temperature of observation 265°C. Specimen put into liquid air.*

standing at that temperature for about an hour. It will be observed that the maximum brightness of the specimen when heated at a given temperature is greater after standing in liquid air and the intensity of the first process is greater. Not much difference is seen in the second process. Curves *A* and *B* show the results of observations made at a temperature of 90°C; curve *C* from observations made at 265°C shows practically the same shape as those taken at the lower temperature. The only difference is that at the higher temperature the luminescence is brighter and takes a longer time to decay. The change in the decay curve of this material due to change in temperature of observation is very different from that observed for other specimens as may be observed from comparison with Fig. 3. Cadmium sulphate has been

found to lose the effect imposed by x-rays very quickly. Material which has stood in liquid air for 30 minutes was observed immediately upon removal from liquid air, then 20 minutes later and was found to have diminished to 0.79 of its intensity in that time. After standing for 24 hours it had diminished to 0.06 of its original intensity. A sample of the same specimen was observed 20 minutes after exposure without having been put into liquid air and it was only 0.59 of the brightness of the one which was put into liquid air and observed immediately after removal. (Temperature of observation in all these cases was 200°C.)

Some samples of sodium chloride plus manganese were found in the laboratory, presumably made for some other experiment. This material was discolored under x-rays from white to light tan, the color becoming greenish when the material was cooled to liquid air temperature. The intensity of thermoluminescence was low but like others it showed two maxima, the second brighter than the first. A decay curve taken from the second maximum showed a straight line, rather an unusual type, and the decay was slow in comparison with that of the sulphates.

EFFECT OF TEMPERATURE OF EXPOSURE

Wiedemann and Schmidt[1] found that calcium sulphates containing manganese absorbed less energy when exposed to cathode rays at high temperatures than when cold as was indicated by less intense thermo-

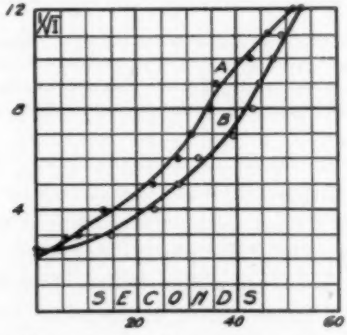


FIG. 5. *Calcium sulphate plus manganese. Decay curves at temperature of observation 110°C.* A—*Exposed at room temperature.* B—*Exposed at higher temperature.*

luminescence. For the sake of comparison some of this material was exposed to x-rays at a temperature above that of the room and the resulting thermoluminescence compared with that of a similar sample

exposed at room temperature. The specimen was mounted about 3¼ inches from the tube in a tinfoil cup placed on the top of a solid copper rod imbedded in hot sand contained in a Dewar flask. The sand was at a temperature of about 200°C at the beginning of the exposure and 40°C at the end. The exposure at room temperature was made with all conditions identical except that the sand was not heated.

Curve *A* in Fig. 5 gives the decay curve of the material exposed at room temperature and curve *B* that of the material exposed at the higher temperature. The maximum brightness of the material exposed at room temperature is greater than that of the specimen exposed at the higher temperature. Although the difference is slight it is nevertheless definite, since it was duplicated on a second trial. This checks the results obtained by Wiedemann and Schmidt. The one exposed at the higher temperature fades more slowly.

EFFECT OF VARIATION OF VOLTAGE OF X-RAY TUBE

With the apparatus used it was possible to vary the voltage in three steps corresponding to 50, 41 and 34 K.V. The current through the tube could not be increased by brightening the filament as it was already

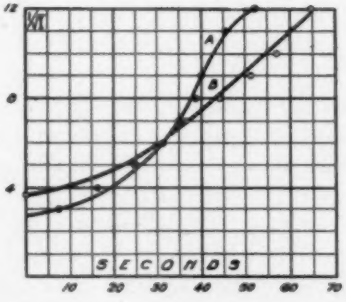


FIG. 6. *Calcium sulphate plus manganese. Decay curves. Material exposed at different voltages.*
A—*50 K.V., 3 milliamperes. Temperature of observation 110°C.*
B—*41 K.V., .75 milliamperes. Temperature of observation 110°C.*

running near its maximum load. A decay curve was taken as indicative of the energy stored. Curve *A*, Fig. 6, is a decay curve after exposure at 50 K.V. and 3 milliamperes. Curve *B* shows that the initial brightness at 41 K.V. and 1.75 milliamperes is less and it takes longer for the light to fade to the lowest intensity easily measurable, than it does after exposure at the higher voltage. It is possible that the low voltage is

sufficient to put into the specimen energy which is emitted in only the first process as the shape of the curve indicates only one process. At 34 K.V. and 0.2 milliamperes the luminescence was too dim for measurement. As seen directly only one process was visible.

QUENCHING OF THERMOLUMINESCENCE

Wiedemann and Schmidt[1] found that the effect of exposure to cathode rays could be quenched by exposure to light. Specimens exposed to x-rays were also found to lose their thermoluminescent property upon standing in the direct rays of the sun for from one to two hours in the middle of the day. Strong sunlight filtered through screens of various kinds was next allowed to fall upon the materials. Exposure was made by placing some of the material on a piece of copper, covering half of it with another piece of copper, then putting the absorbing screen over the whole thing so that the half covered by the copper strip was not exposed to any light and the other half was exposed to that which passed through the filter.

Exposure to the ultraviolet produced a quenching effect in all of the specimens. Red light produced no observable effect. Blue light has a quenching effect in all cases as has also the color of light which exactly matches the color of the luminescence. A quartz spectrograph with an aluminum spark for producing ultraviolet light and a fluorescent screen of anthracene for detecting its presence were used to explore the region of transmission in the ultraviolet of the screens used. In every case those filters which had the quenching effect were found to transmit ultraviolet of a frequency very near the visible. The filters which produced no effect transmitted no ultraviolet.

Wiedemann and Schmidt attributed the quenching effect to the near infrared rays since the effect was found even after the rays had been filtered through a copper sulphate solution or an alum plate. The fact that, as shown by our experiments, heat-absorbing glass and the Uviol glass which transmits ultraviolet were equally effective in destroying the thermoluminescence indicates that the destruction is due to the ultraviolet rather than to the infrared. The results of Wiedemann and Schmidt may be explained by the fact that a copper sulphate solution transmits some of the near ultraviolet.

It seemed reasonable to expect the ultraviolet radiations which produce the quenching effect might do this by speeding up the slow phosphorescence thus causing a more rapid emission of energy much in the same way as a rise in temperature causes a more rapid emission

of energy. A simple experiment was performed to test this. A copper screen with a fine mesh was used to hold the powder under observation upon a screen which was placed in the spectrum of the aluminum spark. Observations upon calcium sulphate plus manganese showed that there was no increase in brightness in the near ultraviolet where the quenching effect lay, nor was there any in the infrared. In the region 2000A to 2400A, however, too short to be transmitted by any of the filters, there was a great brightening, green in color like the thermoluminescence. The specimen which had not been exposed to x-rays did not show this. Exposure of a specimen which had not previously been irradiated by x-rays to this short ultraviolet radiation for 20 minutes produced no thermoluminescence which could be detected by heating even to a higher temperature than that required to bring out the effect due to x-rays. Zinc sulphate plus manganese after exposure to x-rays was found to be luminescent upon exposure to this same region of the ultraviolet. Sodium sulphate and cadmium sulphate were not. Specimens of all of these materials not previously irradiated by x-rays were exposed to the light of the spark both directly and dispersed by a quartz prism so that only wave-lengths 2000A to 2400A would fall on the sample. After such exposure they were not thermoluminescent.

The destruction by one kind of radiation of effects due to another is of common occurrence. Exposure to red and infrared rays will quench phosphorescence of Sidot blende excited by radiations of shorter wavelength. The phenomenon known as photographic inversion is another example of this.[5] A dry plate acted upon by x-rays so that it would blacken on development can have the latent image due to x-rays completely destroyed by exposure to visible light of certain wave-lengths. Villard[6] has attempted an explanation of this effect in terms of the atomic levels involved in the absorption of the radiations.

SUMMARY

The following facts have been observed; (1) Reducing the temperature of the thermophosphors to that of liquid air has no effect upon their subsequent thermoluminescence. (2) Exposure to x-rays at 50,000 volts imparts to each of the substances tested energy which appears as thermoluminescence of two types, one of which decays more rapidly than the other and may be observed for only a short time after exposure if the specimen is allowed to stand at room temperature. The other may

[5] Villard, S. A., *10*, Abs. 1499; 1907.
[6] Villard S. A., *30*, Abs. 1725; 1927.

be observed when the specimen is heated even after it has stood at room temperature for months. (3) Increasing the temperature at which calcium sulphate is exposed decreases the amount of energy stored. The effect is similar to that of cathode rays. (4) Decreasing the voltage at which calcium sulphate is irradiated seems to have the effect of producing only one of the two types of thermoluminescence mentioned above. (5) Exposure to x-rays produces in some samples an ability to fluoresce to the ultraviolet which was lacking in them before exposure. (6) Ultraviolet radiation of frequency very near the visible is capable of completely quenching the ability to thermoluminesce. This effect is not accompanied by any visible increase in the phosphorescence at room temperature. No visible luminescence is excited at room temperature by any part of the spectrum except that between 2000A and 2400A and this is true of these substances only after they have been irradiated.

In conclusion the authors wish to express their appreciation of the generous cooperation of the Department of Physics of Cornell University where the work described was conducted. Thanks are especially due to Professors Nichols and Merritt through whose interest the work was made possible.

F. G. W.,
VASSAR COLLEGE,
POUGHKEEPSIE, N. Y.

M. K. S.,
CORNELL UNIVERSITY,
ITHACA, N. Y.

INSTRUMENT SECTION

A SHIELDED BRIDGE FOR THE MEASUREMENT OF INDUCTANCE IN TERMS OF RESISTANCE AND CAPACITY

By B. W. Bartlett

Since absolute measurements of inductance at speech and carrier frequencies are not generally feasible, inductance standards are usually evaluated by intercomparison with standards of other physical quantities which can be measured accurately in absolute units. For many purposes the measurement of inductance in terms of resistance and capacity is the most convenient and accurate method, as standards of resistance and capacity which are extremely stable over long periods of time and wide ranges of frequency are readily obtainable. A bridge method for the intercomparison of inductance with resistance and capacity has been described by Owen,[1] and has recently been extended by Ferguson,[2] who has worked out a system of electrostatic shielding for the Owen bridge which permits the ready measurement of inductance with a high degree of accuracy.

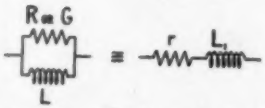


Fig. 1. *Methods of representing the impedance of an inductance coil.*

An attempt to avoid certain difficulties in construction and to obtain a method for making inductance measurements under special circuit conditions led the author to rearrange the impedances of the Owen bridge. This paper describes the bridge thus evolved. It may be completely shielded without undue constructional difficulties and is capable of high accuracy at both speech and carrier frequencies. Among other things it differs from the Owen bridge in that it measures parallel rather than series inductance.

[1] Owen, Proc. of the Physical Society of London, *27*, p. 39; 1914.
[2] J. G. Ferguson, Bell System Technical Journal, p. 1; July, 1927.

Fig. 1 illustrates the two methods of representing the impedance of an inductance coil, either as a pure inductance in series with a pure resistance or as a pure inductance in parallel with a pure resistance. The relations between these inductances and resistances may be written as follows:

$$L=L_1+r^2/\omega^2L_1$$

$$R=r+\omega^2L_1^2/r$$

and

$$G=1/R=r/(r^2+\omega^2L_1^2)$$

In general a bridge using series connections in its arms measures L_1, while a bridge using parallel connections measures L. Obviously as the phase angle of the inductance coil approaches 90 degrees r^2/ω^2L_1 becomes smaller and smaller, and the value of L approaches that of L_1. In the discussion which follows L will be used with the meaning just given.

THEORETICAL DISCUSSION

a. *Equations of Balance.* Fig. 2 shows the circuit of the bridge in simple form. (In this discussion admittances are used rather than impedances because of the mathematical simplicity resulting.) Ideally, the bridge is composed of the pure conductance G_1, the pure capacity

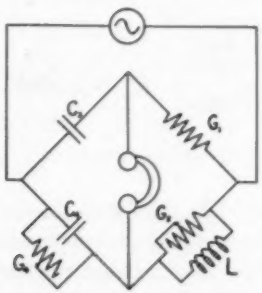


FIG. 2. *Schematic of ideal bridge.*

C_3, the pure capacity C_4 in parallel with the pure variable conductance G_4, and the pure variable conductance G_2 in parallel with the unknown inductance L. The condition for balance is obviously, in symbolic notation,

$$Y_1/Y_3=Y_2/Y_4$$

which may be written

$$\frac{G_1}{j\omega C_3}=\frac{G_2+1/j\omega L}{G_4+j\omega C_4}$$

Whence

$$L=C_3/G_1G_4 \tag{1}$$

and

$$G_2=G_1C_4/C_3 \tag{2}$$

Of course the simple conditions outlined in the preceding paragraph do not obtain in practise. The three conductances G_1, G_2, and G_4 will have a certain amount of susceptance which cannot be eliminated. In the actual bridge the conductances will be shielded resistances and their susceptances will in general be capacitive. In Fig. 3 the first two

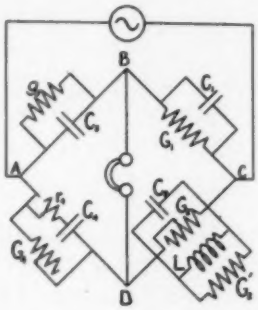


FIG. 3. *Schematic of actual bridge.*

of them are represented by the capacities C_1 and C_2. Since the third is in parallel with C_4 it will be considered part of the latter. The capacities C_3 and C_4 will have slight conductances g_3 and g_4. Finally the unknown inductance L will have a certain conductance G_2'. For simplicity the total conductance of that arm of the bridge will be written g_2, where $g_2=G_2+G_2'$. The condition for balance now is

$$\frac{G_1+j\omega C_1}{g_3+j\omega C_3}=\frac{g_2+j\omega C_2+1/j\omega L}{G_4+g_4+j\omega C_4}$$

Whence

$$L=C_3/(G_1G_4+G_1g_4-g_2g_3+\omega^2C_2C_3-\omega^2C_1C_4) \tag{3}$$

and

$$g_2=G_1C_4/C_3+(G_4+g_4)C_1/C_3-g_3C_2/C_3+g_3/\omega^2LC_3 \tag{4}$$

b. *Inductance.* Equation (3) is, as might be expected, equation (1) with the addition of some correction factors. As their evaluation is not only tedious but extremely difficult as well, some method of eliminating as many of them as possible is desirable. Now it is possible to obtain a so-called zero balance by leaving out the inductance L, cutting out the resistance across C_4, and inserting a small variable resistance r_4 (replaced by its effective conductance g_4 in the formulae which follow) in series with C_4. Both real and imaginary components can be balanced, since there are two independent variables, g_2' and g_4 (g_2' is the value of g_2 at zero balance). The condition for balance is given by

$$\frac{G_1+j\omega C_1}{g_3+j\omega C_3}=\frac{g_2'+j\omega C_2'}{g_4+j\omega C_4'}$$

Whence

$$G_1g_4-g_2'g_3+\omega^2C_2'C_3-\omega^2C_1C_4'=0 \tag{5}$$

and

$$g_2'=G_1C_4'/C_3+g_4C_1/C_3-g_3C_2'/C_3 \tag{6}$$

The left hand member of equation (5) is identical with the correction terms in the denominator of equation (3), except that g_2 has become g_2', C_2C_2' and C_4C_4'. If, therefore, a zero balance is obtained before the bridge is used to measure inductance, the equation for the latter now becomes, by the subtraction of (5) from the denominator of (3)

$$L=C_3/[G_1G_4+(g_2'-g_2)g_3+(C_2-C_2')\omega^2C_3-(C_4-C_4')\omega^2C_1] \tag{7}$$

Now the second, third, and fourth terms in the denominator of (7) can be shown to be negligible compared to the first, G_1G_4. Thus g_3 is the conductance of a good mica condenser, and is evidently of the second order. The subtraction of (4) from (6) permits the evaluation of $g_2'-g_2$, and this can likewise be shown to be a second order term. C_2-C_2' represents the difference in capacity of the corresponding settings of a good resistance box. The difference in these resistance settings is a function of the resistance of the inductance being measured, and for inductances of fairly high time constant such settings will be within a few ohms of each other in several hundred, with the result

that the difference in capacity will be very small. Furthermore as the frequency is increased the difference between the resistance settings decreases. Since the proper choice of C_3 for any given range of inductance and frequency will make $\omega^2 C_3$ of the order of unity, the term $(C_2 - C_2')\omega^2 C_3$ can thus be neglected. The table which follows shows the relative magnitudes of the terms in the denominator of equation (7) under conditions obtaining in practise. While the values given are at a frequency of 1000 cycles they are typical of those up to 50,000 cycles, which was the limit of this investigation. $C_4 - C_4'$, not shown in the table, is the capacity of the conductance G_4, and will not exceed 10^{-10} microfarad. Since C_1 will be even smaller, this last term is quite negligible.

TABLE 1. *Relative Magnitudes of Terms in Equation 7 at 1000 cycles. (Inductance in henrys, capacities in farads, conductances in mhos).*

L	C_3	G_1G_4	$(g_2' - g_2)g_3$	$(C_2' - C_2)\omega^2 C_3$
10.	$.4 \times 10^{-6}$	$.04 \times 10^{-6}$	$.005 \times 10^{-12}$	$.5 \times 10^{-12}$
1.	$.4 \times 10^{-6}$	$.4 \times 10^{-6}$	$.05 \times 10^{-12}$	$5. \times 10^{-12}$
.01	$.04 \times 10^{-6}$	$4. \times 10^{-6}$	$.05 \times 10^{-12}$	$50. \times 10^{-12}$

From the foregoing the error resulting from neglecting the second order terms is seen to be less than .01% in practise. The formula derived for inductance in the ideal case, equation (1), may therefore be used with safety. If instead of conductances the corresponding resistances are used it may be written

$$L = R_1 R_4 C_3 \tag{8}$$

Since R_1 and C_3 are fixed they can be so chosen numerically that L can be read directly in terms of R_4. The bridge can thus be made direct reading for inductance.

c. *Effective Resistance.* The subtraction of equation (6) from equation (4) gives

$$g_2 - g_2' = G_4 C_1 / C_3 + g_2/\omega^2 L C_3 + (C_2' - C_2)g_2/C_3 + G_1(C_4 - C_4')/C_3$$

but

$$g_2 = G_2 + G_2'$$

therefore

$$G_2' = g_2' - G_2 + G_4 C_1/C_3 + g_2/\omega^2 L C_3 + (C_2' - C_2)g_2/C_3 + G_1(C_4 - C_4')/C_3 \tag{9}$$

In the above G_2' is the conductance of the coil, g_2' the conductance of the resistance in the CD arm at zero balance, and G_2 its conductance at bridge balance. The remainder of the terms are correction terms, which cannot be neglected in accurate work. Although it is possible to evaluate them, to do so is tedious, and other methods of obtaining the resistance of the coil, if it is desired, are more expedient.

d. *Shielding.* The advantage of shielding in a bridge of the type used for precision work is that in addition to isolating the bridge from external influences it makes the second order quantities in the bridge equations definite, with the result that they can be allowed for or neglected without introducing any uncertainty into the measurements.

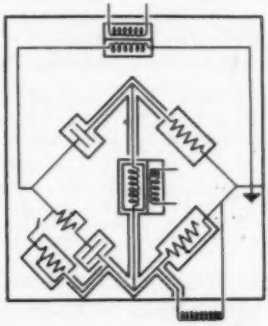


FIG. 4. *Shielding diagram of completed bridge.*

A shielding scheme for the new bridge that is mechanically fairly simple is shown in Fig. 4. It has the advantage over that described by Ferguson[3] for the Owen bridge that it permits one terminal of the inductance to be grounded.

EXPERIMENTAL DATA

A bridge was constructed with the complete shielding just described for measurements at speech and carrier frequencies. As well aged dry-stack condensers of values .4μf and .04μf were available, the other constants were chosen to match them. The resistance R_1 was made up in the form of a one dial box having four settings, 125, 250, 1250, and 2500 ohms. R_2 and R_4 were six-dial shielded resistance boxes having a maximum range of 10,000 ohms. A simple slide wire resistance, about .4 ohm per foot, was used in series with the condenser C_4 for obtaining the zero balance. These constants gave a range of from 10 henrys to

about 5 millihenrys. The transformers were of the type described by Shackelton.[3]

To ascertain the performance of the new bridge under different conditions a series of frequency runs was made on standard air core inductance coils having values of 1 henry, .1 henry, and 20 millihenrys. These coils were then measured on an Owen bridge under conditions as nearly similar to those of the first measurements as possible. The larger coils were also checked on a capacity bridge of the type described by Campbell[4,5] by a resonance method, a source of frequency accurate to at least .005% being available.[6] Table 2 shows the results of these measurements, which covered the speech frequency range. A similar series of measurements was made on shielded 9, 5, and 1 millihenry standards in the carrier frequency range from 10,000 to 50,000 cycles, the results of which are given in Table 3. In this case the check was made only against the capacity bridge.

In general at speech frequencies the new bridge checked the Owen bridge to .03% or better, while both bridges showed a fairly consistent difference of .06% from the capacity bridge. At carrier frequencies the new bridge checked the capacity bridge to better than .1%. When the former was used without the variable zero-balancing resistance in series with C_4 errors as large as .3% resulted even at speech frequencies, showing the importance of the second order quantities in the bridge equations.

DISCUSSION OF RESULTS

The theoretical discussion of the bridge leads to the conclusion that its accuracy depends on the extent to which equation (5) holds true. The accuracy of this equation depends on the precision with which a zero balance can be obtained. A good slide wire resistance in series with C_4 can be set to .001 ohm at 1000 cycles. Accordingly g_4 will be correct to $\pm 10^{-9}$, and since the maximum value of G_1 is of the order of 8×10^{-3} any error resulting from inaccuracy in the zero setting will be less than .005%. As the frequency increases the precision with which g_4 can be set also increases, so that the inaccuracy from this cause will not increase appreciably in the carrier range.

[3] W. J. Shackelton, A.I.E.E. Journal, Feb. 1927.
[4] G. A. Campbell, Electrical World and Engineer, April 2, 1904.
[5] G. A. Campbell, Bell System Technical Journal, p. 18; July, 1922.
[6] J. W. Horton, N. H. Ricker, W. H. Marrison, A.I.E.E. Trans., June, 1923.
For a general discussion of types of inductance bridges see Hague, Alternating Current Bridge Measurements, Pitman, 1923.

TABLE 2. *Speech frequency measurements of inductance.*

Frequency in cycles	Owen Bridge	Difference in %	New Bridge	Difference in %	Capacity Bridge
	Inductance in henrys		Inductance in henrys		Inductance in henrys
1000	1.0073	.00	1.0073	.06	1.0079
2000	1.0141	.03	1.0138	.06	1.0144
3000	1.0256	.03	1.0253		
1000	.10065	.00	.10065	.05	.10070
2000	.10127	.01	.10126		
3000	.10229	.00	.10229	.06	.10235
1000	.019995	.005	.019996		
2000	.019998	.02	.019994		
3000	.020003	.005	.020004		

Note: Although the Owen bridge measures values of L_1, for purposes of comparison they have been reduced to values of L in this table by means of the formula relating L and L_1.

TABLE 3. *Carrier frequency measurements of inductance.*

Frequency in cycles	New Bridge	Difference in %	Capacity Bridge
	Inductance in millihenrys		Inductance in millihenrys
10,000	9.063	.04	9.067
20,000	9.152	.05	9.147
30,000	9.301	.09	9.292
40,000	9.516	.09	9.507
50,000	9.811	.15	9.796
10,000	5.035	.03	5.029
20,000	5.055	.02	5.051
30,000	5.113	.15	5.083
40,000	5.165	.09	5.147
50,000	5.240	.08	5.224
10,000	.9989	.02	.9987
20,000	1.0010	.06	1.0004
30,000	1.0050	.15	1.0035
40,000	1.0102	.03	1.0099
50,000	1.0175	.15	1.0160

Practically the accuracy of bridges of this type depends on the accuracy with which the values of the resistance and capacity standards are known. Hence in this case both the new bridge and the Owen bridge should be accurate to $\pm .05\%$ and consistent to $\pm .02\%$ at speech frequencies, since the capacity standards were known to better than $\pm .03\%$ and the resistance standards to better than $\pm .02\%$, and both were consistent to $\pm .01\%$. At carrier frequencies the capacities were known only to $\pm .05\%$, and the accuracy of the bridge decreases in proportion. The experimental results in Tables 2 and 3 lie within these values. At speech frequencies the new bridge and the Owen bridge are in general agreement to .02%, and check the capacity bridge to about .06%, while at carrier frequencies the former checks the capacity bridge to .1%. That the latter figures are consistent with the statement that the absolute accuracy of the new bridge is $\pm .05\%$ may be shown as follows.

The formula for inductance on the capacity bridge may be written $L = a/Cf^2$, where a is a constant, C the capacity, and f the frequency. The formula for the new bridge has the form, $L = bRC$, where b is another constant and R the resistance. By one method inductance is directly proportional to capacity and by the other inversely so. Thus, since the possible errors in resistance and frequency are negligible in comparison, the difference in the results by the two methods will evidently be twice the absolute error in the value of the capacity standards, provided the latter are consistent within themselves, which is the case.

The new bridge as constructed had a range from 10 to .005 henrys. This range includes the majority of inductances encountered at speech frequencies, and can be raised somewhat by increasing the capacity of the condenser C_3. The lower limit, being a function of the ratio of resistance to reactance in the coil under measurement, decreases with increasing frequency; hence in the carrier field, where in general the inductances to be measured are smaller, its range will keep pace with the lowering of inductance values.

In the case of Ferguson's arrangement of the Owen bridge the inductance being measured is partially grounded through a resistance. This is variable, with the result that the coil is measured under indefinite conditions with respect to its distributed capacity to ground. In the new bridge, since one terminal of the coil is grounded, this condition is always perfectly definite. In the case of coils of large physical

size this difference in the conditions of measurement may result in considerable difference in the results.

CONCLUSIONS

The bridge just described will measure inductance at frequencies up to 50,000 cycles with an accuracy equal to that of any bridge method now in use. This accuracy is limited by the accuracy with which standards of capacity and resistance can be obtained. In the present case it was $\pm .03\%$ at speech and $\pm .05\%$ at carrier frequencies. A method of making the bridge direct reading and a complete shielding scheme permitting measurements to be made with one terminal of the inductance grounded are worked out and discussed.

In conclusion I wish to express my indebtedness to the Bell Telephone Laboratories, where the experimental work was done, and in particular to Mr. J. G. Ferguson, at whose suggestion it was initiated.

B. W. BARTLETT

BOWDOIN COLLEGE,
BRUNSWICK, ME.,
FEB. 20, 1928.

THE "ULTRAMICROMETER", A NEW INSTRUMENT FOR MEASURING VERY SMALL DISPLACEMENT OR MOTION, AND ITS VARIOUS APPLICATIONS[1]

By Juichi Obata

ABSTRACT

In the present article are described the detailed construction and various examples of applications of the "Ultramicrometer," a new instrument for measuring a small displacement or motion utilizing a generating valve circuit. In its usual form the instrument consists of a tuned grid circuit, but it can easily be changed to the Colpitts-Hartley circuit. The displacement or motion to be measured is made to produce either a change in the capacity in the circuit or a change in the eddy-current loss, and in consequence a corresponding change in the anode current of the valve results. This change in the anode current is measured with a galvanometer, or if a permanent record is needed, an Einthoven string galvanometer or a Duddell oscillograph is employed. The anode current itself is balanced by means of a potentiometer method, or a small transformer is inserted in the anode circuit.

With this instrument the capacity method as well as the eddy current method may be carried out, and the anode current may be balanced or a transformer may be inserted according to the kind of problem; all the necessary resistances and transformer being contained in the instrument.

As the examples of application of the instrument are described the recording of earth-movements, direct recording of accelerations, an electrical indicator for high speed internal combustion engines, precise recording of sounds, pressure variations and various mechanical vibrations.

1. INTRODUCTION

A couple of years ago the present author[2] devised a new instrument for measuring small displacement or motion utilizing a generating valve circuit. Some improvements in design were thereafter made, and the instrument has since been employed for various kinds of problems with satisfactory results, and in consequence the author possesses now a fair degree of confidence about the efficient applicability of the instrument on diverse fields of physical and technical measurements. Therefore, it may not be out of place here to publish the detailed construction of the instrument and the results of its various applications.

2. THE PRINCIPLE OF WORKING AND THE CONSTRUCTION OF THE INSTRUMENT

To measure a displacement or motion of a body by means of this instrument, two different methods can be carried out. In both cases

[1] The present instrument is supplied by Messrs. Andow & Co., No. 1359, Nakamaru, Himonya, Hibusuma-machi, Tokyo-fu.

[2] J. Obata and Y. Yosida; "On the valve method of measuring small motion, with special reference to the precise recording of sounds, pressure-variations and vibrations." Report of the Aeronautical Research Institute Tokyo Imperial University, *1*, No. 11; 1925.

an electrical oscillation, having a frequency of some 600,000 cycles per second, is started in a circuit containing a thermionic valve, and the displacement or motion to be measured is made to produce a corresponding change in the anode current of the valve. This change in the anode current is measured or recorded by means of a galvanometer or an oscillograph. The novel feature of these methods lies in their extreme sensitivity, without requiring anything being brought in contact with the moving body.

A number of papers have already been published by Whiddington,[3] Dowling,[4] Thomas[5] and many others relating to the method of measuring very small motion utilizing one or several generating valve circuits. The method devised by Whiddington utilizes the beat between two generating valve circuits and is the so-called "heterodyne" method. On the other hand, Dowling and Thomas work with only one generating circuit, and the displacement or motion to be measured is made to produce a corresponding change in the anode current of the thermionic valve. In the Dowling's method this change in the anode current is caused by the change in a capacity in the circuit, while in the Thomas method it is produced by the change in the eddy-current loss. Thomas devised an instrument for measuring small motions utilizing the said principle, and in devising the present instrument the author is indebted in some respects to the Thomas' design.

In the present instrument use is made of only one generating valve circuit, and it is so constructed as to be capable of carrying out both these methods, i.e the capacity method as well as the eddy-current method.

(*i*) *The capacity-change method.* In its usual form the instrument consists of a tuned grid circuit as shown in Fig. 1, (in this case *M* has no bearing with the circuit) but it can easily be changed to the Colpitts-Hartley circuit. In a valve circuit as shown in Fig. 1, an electrical oscillation, having a frequency of about 600,000 cycles per second, is started by changing the total number of turns of the oscillation coil *S* as well as the number of turns of the part of the same coil contained in the grid circuit. These adjustments are made by means of the sliding contracts (a) and (b).

(*ii*) *The eddy-current method.* A similar generating valve circuit is utilized. Referring to Fig. 1, *M* is the body, the displacement or

[3] R. Whiddington: Phil. Mag., *40*, p. 634; 1923; *46*, p. 607; 1923; *49*, p. 113; 1925.
[4] J. J. Dowling: Engineering, 395; 1921. Phil. Mag., *46*, p. 81; 1923.
[5] H. A. Thomas: Engineer, *135*, p. 138; 1923. Journ. Sci. Instr., *1*, p. 22; 1924.

motion of which is to be measured. A change in the distance between M and the oscillation coil S gives rise to a change in the eddy-current loss and in consequence a change in the anode current is resulted. If the moving body is not metallic, a small sheet of tin-foil pasted on the surface of the body is sufficient to cause the eddy-current loss.

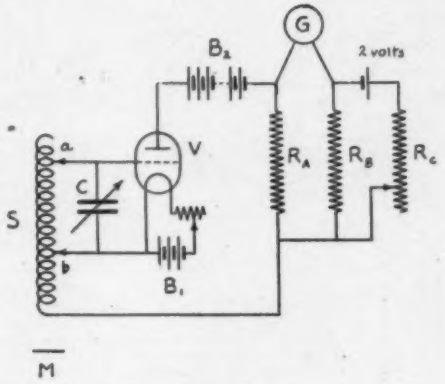


FIG. 1.

In carrying out any of these methods, it is the change in the anode current and not the anode current itself, that we want to measure. The latter should, therefore, be balanced by means of a potentiometer

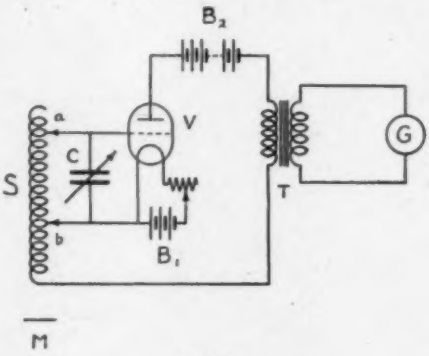


FIG. 2.

method as shown in Fig. 1 or if vibratory motions having high frequencies are concerned, it is much more convenient to insert a small transformer in the anode circuit as shown in Fig. 2. All the necessary resistances and transformer are contained in the instrument.

In the case of measuring a single definite displacement a usual galvanometer, with a suitable shunt, is employed as *G*. If a permanent record is required, an Einthoven string galvanometer is to be employed.

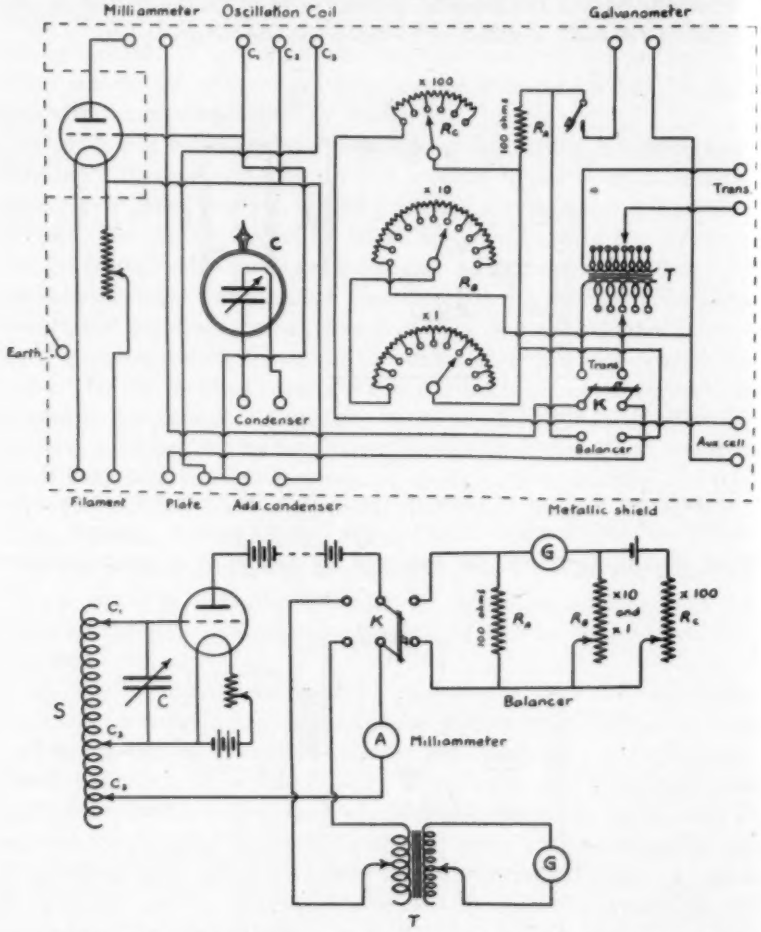
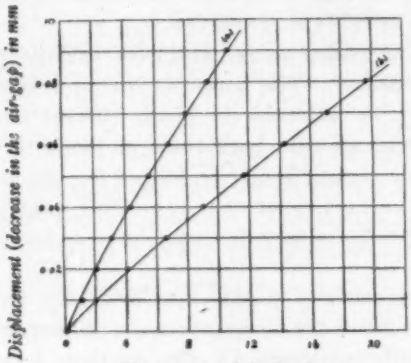


FIG. 3. *The connection diagram of the instrument.* (*In the latest form of the instrument, the contact C_3 is fixed, and contacts C_1, C_2 are movable.*)

or a Duddell oscillograph may also be employed as the recording instrument by amplifying the change in the anode current.

(*iii*) *The construction of the instrument.* The connecting diagram and detailed construction of the instrument are shown in Fig.3, and

the photograph of the instrument is reproduced in Fig. 9. All the necessary resistances and transformer are contained in a wooden box, the inner side of which is lined with a sheet of grounded metal. By means of a switch K the balancing resistances or the transformer may be inserted in the circuit.



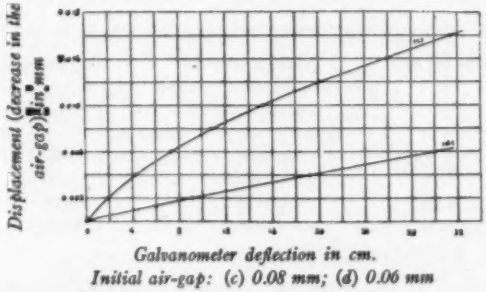


FIG 4. *The sensitivity curves. Apparatus employed: Valve: "Radiotron" 201 A. Galvanometer: Leeds and Northrup Co., wall type galvanometer, (shunted with 20 ohm resistance). Scale distance: 180 cm. Parallel plate condenser: Circular plates, dia. 5 cm. The displacement was produced by means of the focusing device of a Zeiss microscope.*

The oscillation coil S is mounted separately on a stand and is made capable to move axially by means of a precision micrometer. Thus, in carrying out the second method, i.e. the method utilizing the eddy-current loss, the calibration may easily be made *in situ* by changing the relative position of the coil with respect to the moving body.

In carrying out the capacity-change method, a simple variometer such as used in a radio receiving set may be employed as S.

(*iv*) *The sensitivity.* At first sight it would appear, that by employing a sensitive galvanometer to measure the change in the anode current the sensitiveness of these methods may be increased to the utmost degree, say, such as to be able to measure a minute displacement of the order of 10^{-8} cm or still less.

As a matter of fact, such an extraordinary sensitivity cannot, however, be easily obtained. The reason is that it is a matter of considerable difficulty to maintain the anode current unchanged for a long period of time, while in both these methods the extreme constancy of the anode current is *a priori* looked for. In order, therefore, to keep the anode current as constant as possible, the filament battery B_1 as well as the anode battery B_2 should have a capacity as large as possible.

Obviously, the sensitivity of the capacity-change method depends upon the initial width of the air-gap between the parallel plates. Fig. 4 shows the sensitivity curves, i.e. the relations between displacement and the galvanometer-deflection for four different air gaps.

In the following table are given the mean sensitivity for different initial air-gaps.

Initial air gap	Mean sensitivity (Displacement causing 1 mm galva.-defl.)	Magnification
0.5 mm	0.0007_7mm	1300
0.3	0.0003_7	2700
0.08	0.0000_4	25000
0.06	0.0000_2	50000

The sensitivity of the capacity-change methods is thus very high, although a special parallel plate condenser should be built for each case. It is, therefore, very suitable for measuring small displacements, minute vibrations, elongations etc., as will be shown later.

On the other hand, the second method, i.e. the method utilizing the eddy-current loss is somewhat inferior to the first method with respect to the sensitiveness. It is, however, very convenient in some cases and has a wide field of applications, because no special apparatus is required in carrying out this method. All that is required is to bring the oscillation coil S at a small distance, usually a few millimeters,

from the vibrating body. Further merits of this method are that the calibration can be made *in situ* and the wider working range compared with the capacity-change method.

As already stated, it is not easy to measure with this instrument a minute movement occuring in a long time, since it is considerably difficult to maintain the anode current constant for a long period of time. It may, therefore, be well said, that the present instrument will exhibit its true efficiency in recording minute vibrations, rather than in measuring a small definite displacement. Various such applications will be given.

3. VARIOUS APPLICATIONS OF THE INSTRUMENT

The present instrument has been employed for a number of problems and proved very satisfactory. The details having been published in each case, only a brief description will be here given.

(A) THE CAPACITY-CHANGE METHOD.

(i) The measurement of pulsatory oscillations, microtremors and tiltings.[6]

The extreme sensitiveness of the method will soon make one notice that the instrument may justly find a good application in the measurement of pulsatory oscillation or as a sensitive seismograph.

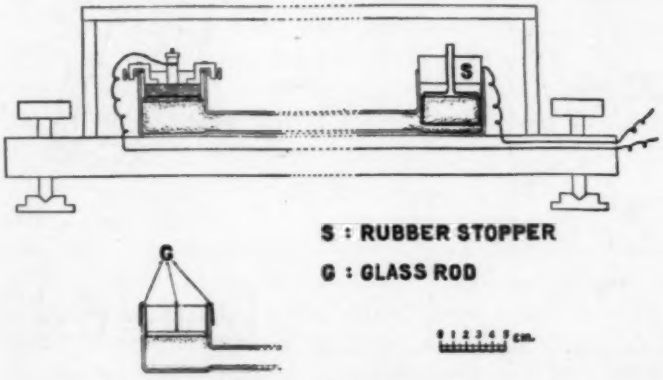


FIG. 5. *Apparatus used for recording earth movements.*

The electrical connection as shown in Fig. 1 is employed. A long glass tube like a flat U form was held horizontally and filled with mercury as shown in Fig. 5. The parallel plate condenser C was thus

[6] J. Obata: Proc. Phys.-Math. Soc., Japan, *9*, p. 1; 1927.

formed between the surface of a thin steel plate floating on the surface of mercury and a steel plate fixed on the edge of the glass cup. The minute up-and-down movement of the mercury surface due to the tilting of the earth-crust produces a change in the capacity, and is recorded as already described.

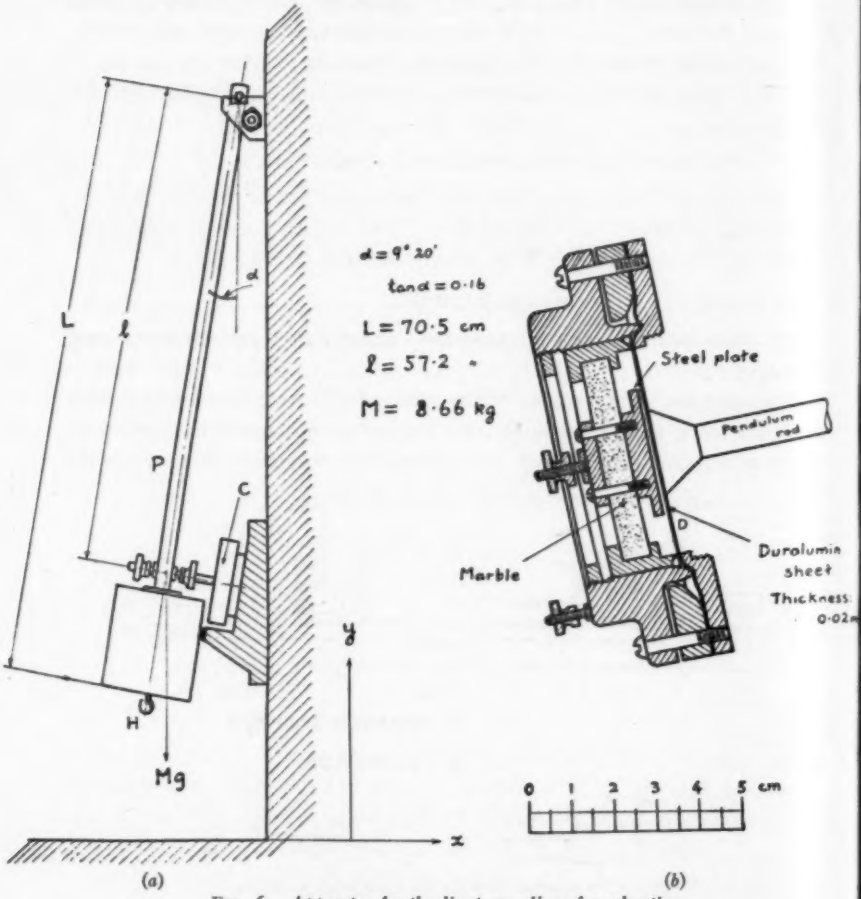


FIG. 6. *Apparatus for the direct recording of acceleration.*

Some of the typical records obtained with an Einthoven string galvanometer are reproduced in Fig. 10, in which (a) and (b) are the records obtained with mercury tubes two- and one-meter long respectively.

Arrangement using a single mercury cup and that using a horizontal pendulum were also experimented with; the original paper should be referred to for the details.

(*ii*) *An electrical device for the direct recording of acceleration.*[7] The direct determinations of accelerations by means of an electrical device was already carried out by B. Galitzin and H. O. Wood, utilizing the piezo-electricity of quartz or Rochelle salt. The acceleration to be measured was made to produce a change in pressure applied to the crystal and the potential difference exerted piezo-electrically was recorded either with a string electrometer or a Duddell oscillograph. As the present instrument may be employed for recording a minute change in pressure, it may also be applied to the direct determination of acceleration.

The apparatus is shown in Fig. 6 (a) and consists essentially of a steel pendulum P and a condenser microphone C. The detailed construction of the latter is shown in Fig. 6 (b). D is a highly stretched diaphragm of duralumin, 0.02 mm thick, and it forms a parallel plate condenser with a steel plate separated from it by a narrow air gap of about 0.03 mm. Thus a change in pressure applied upon the duralumin membrane by the pendulum causes a corresponding change in the capacity of the condenser, which, in turn, is made to produce a deflection of a galvanometer using an electrical arrangement as shown in Fig. 1. The membrane being very tightly stretched, no appreciable displacement is given rise in any part of the system.

Using the lettering of the various parts as shown in Fig. 6(a), the relation between the acceleration $\ddot{x}$ and the deflection u of an Einthoven string galvanometer is given by

$$\ddot{x} = u/kM \cos \alpha,$$

where k is a constant.

In the apparatus constructed by the author extremely high sensitivity could be obtained using an iron mass of 8.6 kg and only one simple circuit as shown already; 1 mm galvanometer-deflection being corresponded to an acceleration of 0.4 cm per second[2] or about (1/2500)g.

(*iii*) *An electrical indicator for high speed internal combustion engines.*[8]

The novel feature of the indicator is that it contains no mechanical part which exhibits appreciable friction or inertia, and moreover,

[7] J. Obata: Proc. Phys.-Math. Soc., Japan, *9*, p. 83; 1927.

[8] J. Obata and Y. Yosida: Report of the Aeron. Research Inst., Tôkyô Imperial University No. 28; 1927; Engineering p. 253; Aug. 26, 1927.

it is entirely free from the influence of the natural vibration of any part of the system, so that it is especially suited for high speed engines, with which ordinary indicators fail to give correct diagrams. The indicator is nothing but the well known disc form. The only difference from the familiar mechanical or optical ones lies in the use

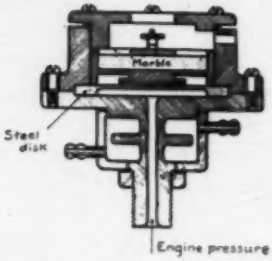


FIG. 7.

of a very thick steel disc, 2 mm thick and 5 cm in diameter, and of an extremely sensitive electrical means here dealt with, i.e., the "ultra-micrometer method," for recording the minute motion of the thick disc caused by the pressure in the cylinder. The details of construction of the apparatus is shown in Fig. 7. A parallel plate condenser is formed

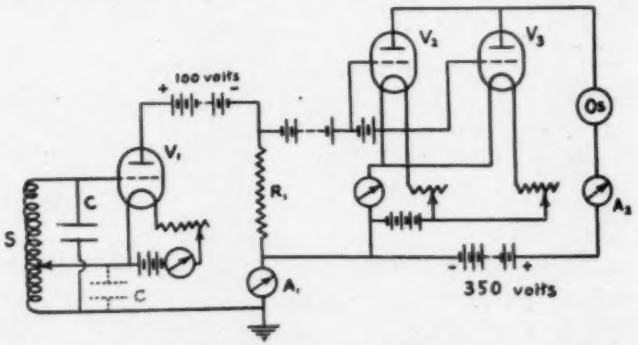


FIG. 8. *Electrical arrangement for the indicator.*
The oscillation is started more easily by inserting the condenser C (indicator) in the position shown with dotted line.

by the disc and a steel plate placed at a very small distance from the disc and electrically insulated from it, and the condenser is employed as usual as a capacity in a generating valve circuit. In Fig. 11 is reproduced a record of the engine-pressure of an "Indian" motor-

cycle engine obtained with an oscillograph. The electrical arrangement employed is shown in Fig. 8; the change in the anode current was further amplified by means of large valves. A string galvanometer or a string electrometer may also be employed.

(*iv*) *The recording of sounds, pressure-variations and vibrations.*[9] A condenser microphone may justly be employed as the capacity in the present instrument. The present author has constructed a condenser microphone using a sheet of German silver, 0.03 mm thick, or of duralumin, 0.02 mm thick. Using a Duddell oscillograph several records of sounds were obtained.

Further, the condenser microphone, if slightly modified, may easily be employed as a precision pressure gauge. Records of pressure change thus obtained with a string galvanometer will be found in the Report 11, already referred to. As a simple pressure recorder a much more simple construction than the condenser microphone will be sufficient.

(B) THE EDDY-CURRENT LOSS METHOD

As already stated, the method utilizing the eddy-current loss requires nothing to be specially constructed, so that it is very convenient for recording vibrations.

Records of vibrations thus obtained by the present author are given in the Report already referred to.

In Fig. 12 are reproduced further examples of such records of vibrations. Namely, (a) is the vibration of a turbine blade recorded with a string galvanometer, (b) is the similar vibration recorded with an oscillograph, and (c) is the vibration of a wooded bar, clamped at one end, recorded with an oscillograph.

These beautiful records were obtained by Mr. H. Takemoto, Engineering College, Kyûshû Imperial University, employing the present instrument, and reproduced here with his kind permission.

In conclusion the author's thanks are due to Mr. Y. Yosida for his able assistance and also to Messrs. Andow & Co., for courtesy and effort offered throughout the course in developing the present instrument.

PHYSICS DIVISION,
AERONAUTICAL RESEARCH INSTITUTE,
TÔKYÔ IMPERIAL UNIVERSITY,
TÔKYÔ, JAPAN.

[9] J. Obata and Y. Yosida: Report Aeron. Research Inst., No. 11; 1925.
J. Obata: "Discrimination of the Quality of Brick by means of Sound." Journ. Franklin Inst., *203*, p. 647; 1927.

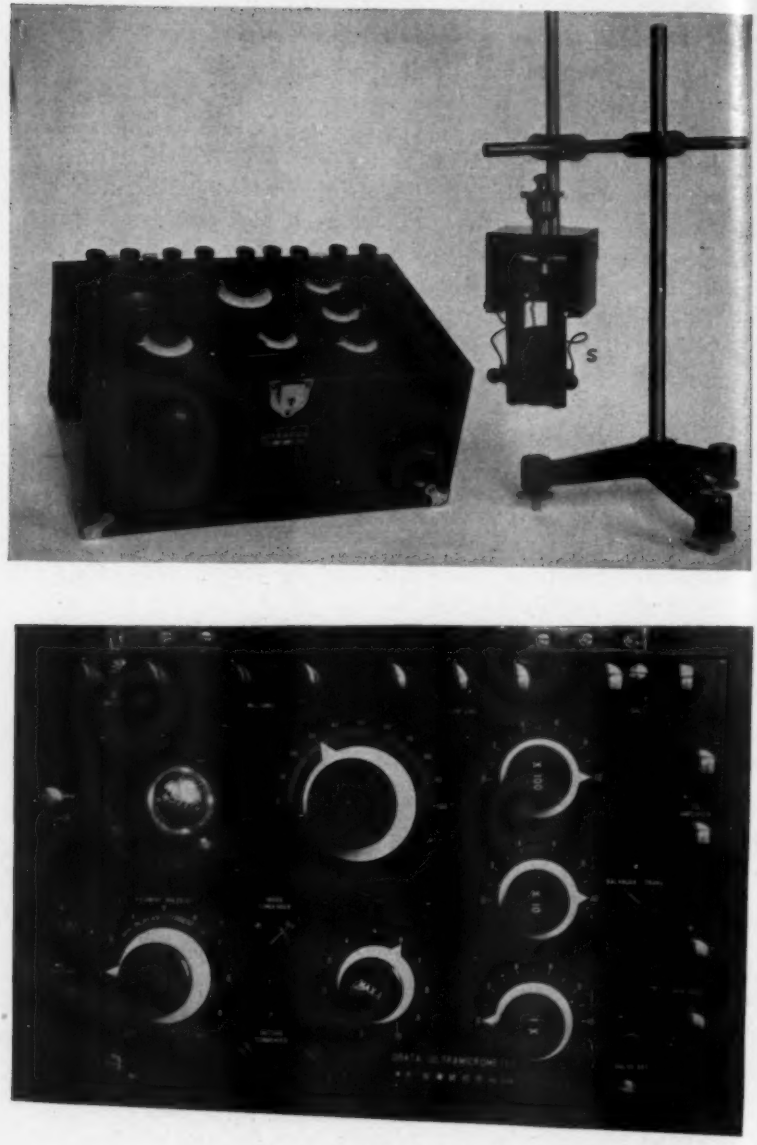


FIG. 9. *The ultramicrometer.*

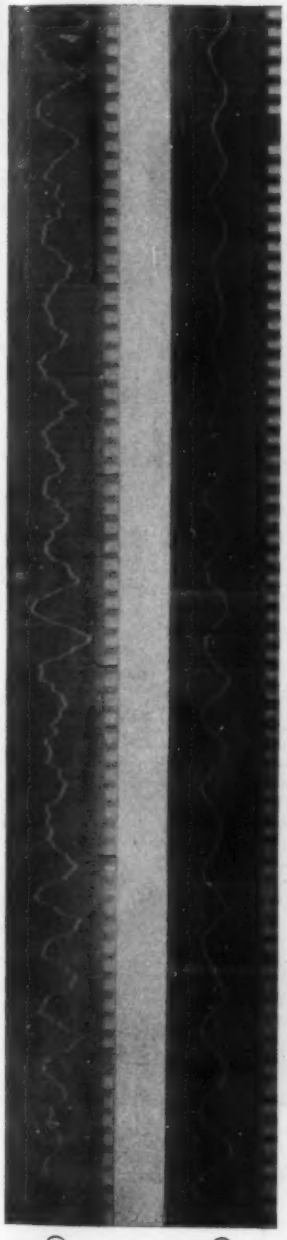


FIG. 10. *Records of earth-movements, obtained with a string-galvanometer.*

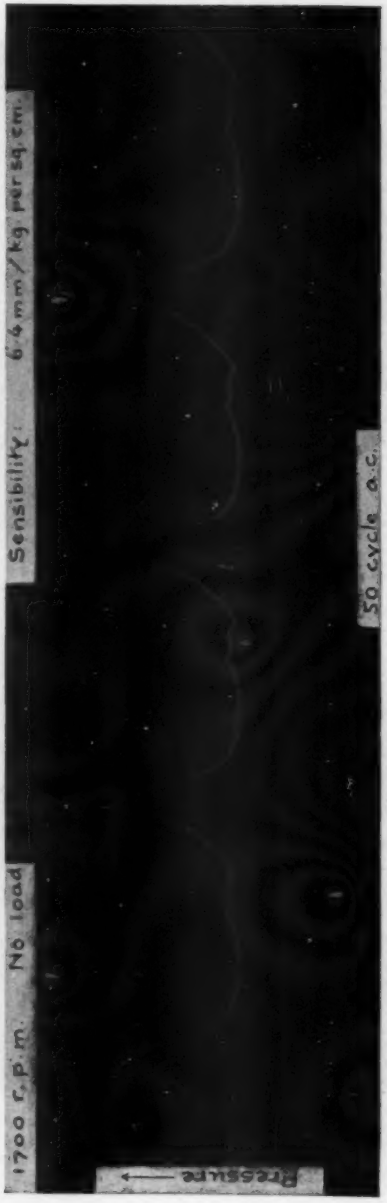
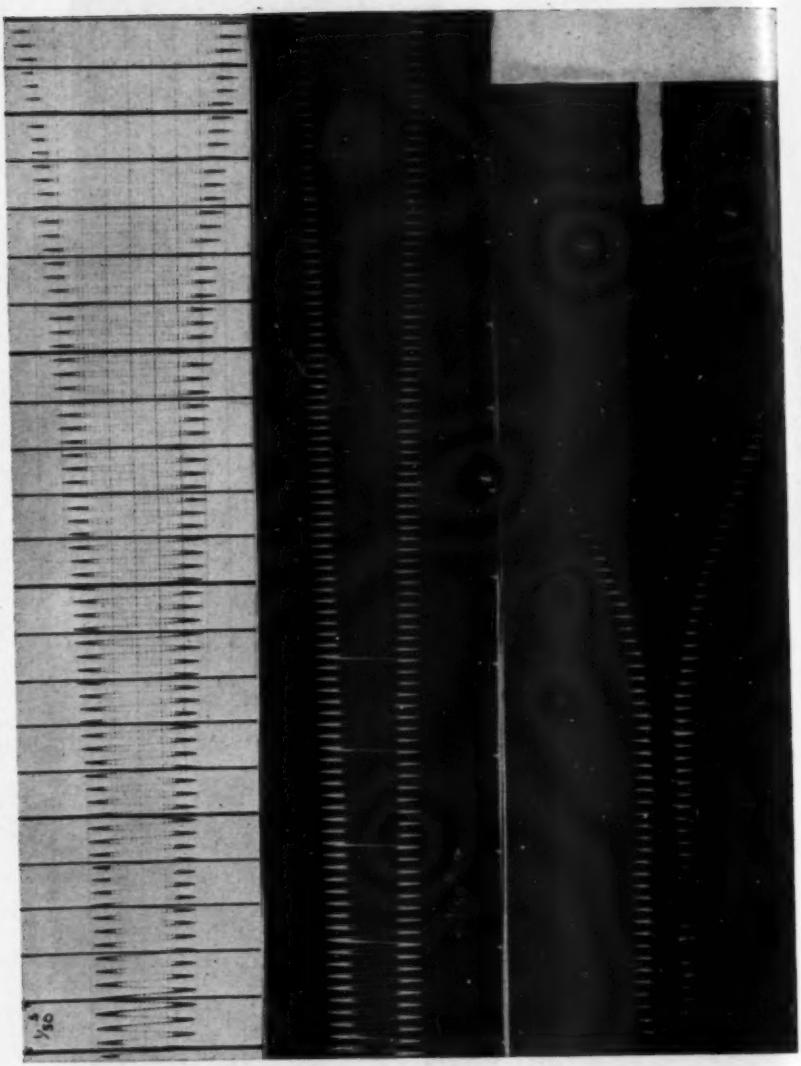


FIG. 11. *Pressure record of the Indian motor-cycle engine at 1700 r.p.m., obtained with a Duddell oscillograph.*



(a) *Vibration of a turbine blade recorded with a string galvanometer.*

(c) *Vibration of a wooden bar (one end clamped).*

(a) (b) (c)

(b) *Vibration of a turbine blade recorded with an oscillograph.*

A NEW METHOD OF SPECTROSCOPY FOR FAINT X-RADIATIONS

By P. A. Ross

In the study of scattered x-rays the extreme faintness of the radiation makes satisfactory analysis by the ordinary crystal reflection method extremely difficult. The method here described[1] seems to have a field of usefulness where such faint sources are to be dealt with. It may enable the investigator to deal with radiations more than a thousand times fainter than could be analyzed by the crystal method.

In brief, this method consists in using two adjacent elements (silver-cadmium, palladium-silver, platinum-gold, etc.) in the path of the x-ray beam. The thicknesses of the two foils are so adjusted that the transmission curves of the two coincide except between their K discontinuities. These balanced foils are mounted on a carrier in front of the window of an ionization chamber so that either foil may be moved into exact position in front of the window at will. The increase in the ionization current, when the metal of higher atomic number is substituted for the one of lower atomic number, is due to the band of radiation lying between their K absorption limits.

PROCEDURE

Two pieces of chemically pure silver and cadmium were rolled to a thickness of about .04 mm and about 3 cm by 6 cm in dimensions. Each foil was stretched in a small rectangular frame moving on guides between adjustable stops so as to insure exact return to the same position in the x-ray beam. Tangent screws were provided for rotating these frames about a vertical axis so that the obliquity of the plane of the foil to the x-ray beam might be varied, thus changing the effective thickness and achieving balance with the other foil (Fig. 1).

An ordinary ionization spectrometer was used to determine the transmission curves and check their coincidence at various points A typical balanced transmission curve is shown in Fig. 2.

After the absorption foils had been properly balanced the *crystal was removed* from the spectrometer table, the ionization chamber swung into direct line with the slits, and the slits opened to several millimeters width. Ionization current readings were taken with the *foils alternating*

[1] P. A. Ross, Phys. Rev., *28*, p. 425; 1926.

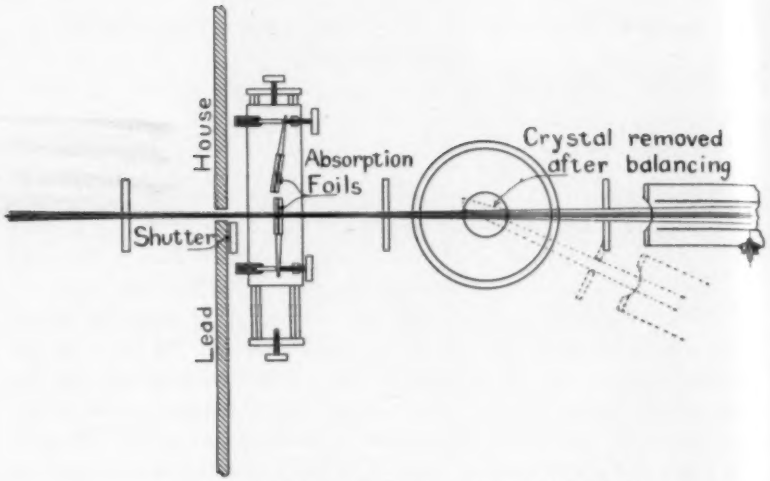


FIG. 1. *General scheme of apparatus for balanced foil method.*

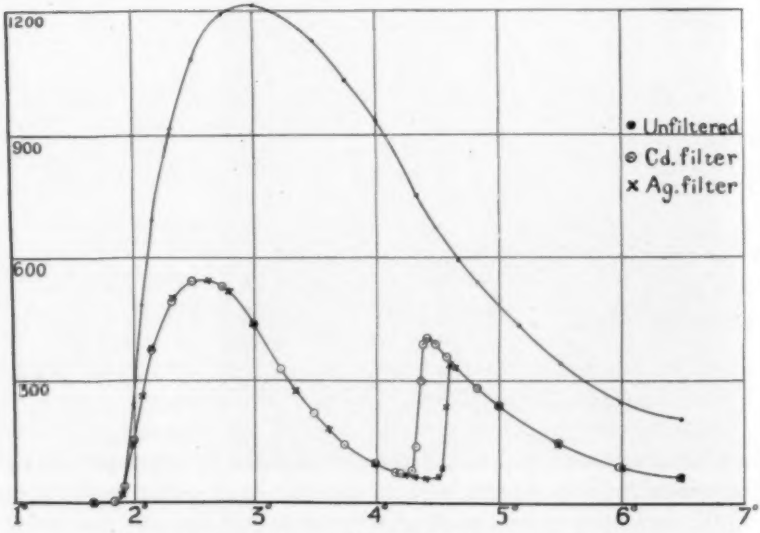


FIG. 2. *Balancing curves. Intensity plotted against crystal angles.*

in the path of the beam and their *differences* recorded as representing the intensity of the band of radiation between their K limits. Usually isochromats were run with each spectrum band so isolated. In the case of cadmium-silver the width of this band was about .022A and the mean wave-length .474A. For lead-bismuth the band width would be .0037A with a mean wave-length of .1383A.

The intensity was nearly uniform across the band. Furthermore the balance of the foils and consequent purity of the band of radiation dealt with was independent of the width of the slits over a very great range. An important point also is the complete absence of second order radiation.

AVAILABLE ELEMENTS FOR THIS METHOD

A canvass of the metallic elements that are available in foils shows that while many pairs of adjacent elements may be obtained, there are large gaps in some of the most interesting regions of the spectrum. This method could be applied in such regions only by the use of salts. The general absorption of the undesirable constituents of the chemical compound decreases the height of the K discontinuity relative to the general absorption, thus reducing the efficiency of the method. Furthermore (and far more serious), the undesirable portion of salt A makes balance with salt B impossible by giving an absorption curve of different form, unless the undesirable portions of the two salts are chemically identical and present in equivalent amounts. This has led to some partially successful attempts to add properly chosen materials to the salts so that the general absorption curves would coincide. Mr. C. S. Barrett[2] has succeeded in doing this much better than the writer. BaO may be balanced against CsOH quite successfully and in general the difference between oxides and hydroxides seems to be negligible. Very fortunately the rare earths all show the same valence. Work with rare earth salts borrowed from the University of Illinois, through the kindness of Professor B. Smith Hopkins, has shown that their like salts may be very accurately balanced.

COMPARISON WITH CRYSTAL METHOD

Bergen Davis[3] has shown that a good cleaved calcite crystal may have a reflection efficiency, r, as high as 40% or 50% for monochromatic radiation striking the crystal face within a small angle, about 10″, of the angle of reflection required by Bragg's law, $\lambda = 2D \sin \theta$. For

[2] C. S. Barrett, Phys. Rev., *31*, p. 159A; 1928.
[3] Bergen Davis, Phys. Rev., *17*, pp. 608–623; 1921.

rays deviating from θ by an amount greater than a few seconds the efficiency of reflection becomes practically zero.

Let us assume that the x-ray spectrometer has two slits of equal width at such a distance apart that the angular deviation within the beam shall be $\alpha = 10'$. Let us also assume a uniform distribution of energy over the portion of the source visible from the crystal. Richtmyer[4] has shown that under such conditions any particular wavelength in the reflected ray would give a triangular peak when intensity

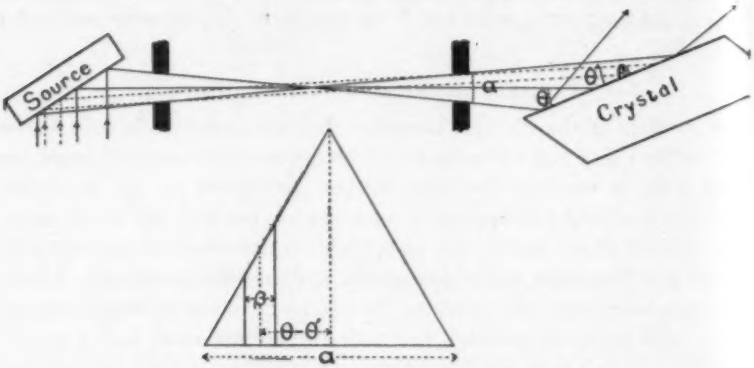


FIG. 3. *Inefficiency of crystal when used with wide slits.*

is plotted against angular width of the reflected beam. (Fig. 3.) Then the efficiency of reflection for monochromatic radiation in this case would be

$$R = \frac{r\beta}{\alpha}\left(1 - \frac{\theta - \theta'}{\alpha}\right)$$

where R = reflecting efficiency with monochromatic radiation and large slit width α.

r = reflecting efficiency with monochromatic radiation and very small slit width.

θ = angle between center line of slits and crystal face required by Bragg's law for the wave length used.

θ' = actual setting of the crystal.

α = angular slit width used.

If "white" radiation instead of monochromatic be used the above expression becomes

[4] F. K. Richtmyer, Phys. Rev., *26*, pp. 724–735; 1925.

$$R' = r\beta/2\alpha.$$

As a numerical example let us take $r = .5$ when $\beta = 20''$ for cleaved calcite. For purposes of comparison let us make the slit width $\alpha = 15'$ so that the spectrum width of the band of radiation reflected by the crystal shall be about the same as that isolated by the foil absorption method, i.e., about .022A. Then

$$R' = \frac{.5 \cdot 20''}{2 \cdot 900''} = \frac{1}{180} \text{ (efficiency of crystal reflection)}.$$

The efficiency of foil transmission may easily be made .5 (Fig. 2) or more giving the foil an advantage of at least 90 fold for the same degree of monochromatization and slit width. Since the foil method suffers no loss of efficiency or increase of spectral band width with any reasonable increase in slit width, and since for most scattered radiation work on polarization or Compton effect, a deviation of scattering angle from the mean amounting to 2° or more may easily be tolerated, the advantage of the foil method may be increased to more than a thousand fold without decrease of "resolving power."

$$\text{"Advantage"} = \frac{\text{(efficiency of foil's at any width)} \cdot \text{(actual s'it width)}/15'}{\text{(efficiency of crystal at } 15' \text{ slit width)}}$$

$$= \frac{.5 \cdot 240'/15'}{1/180} = 1440.$$

On page 375 of this issue the writer has published a brief preliminary paper on the polarization of x-rays from a standard tungsten target Coolidge tube. This serves to illustrate the application of the foil method to at least one type of research problem.

DEPARTMENT OF PHYSICS,
STANFORD UNIVERSITY,
CALIFORNIA.

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Through the generosity of the Association of Apparatus Makers of the United States, the Journal of the Optical Society and Review of Scientific Instruments announces a prize of $250.00 for the best paper on Scientific Instruments and methods, other than optical, published in the Instrument Section of this Journal during 1928. The Journal publishes in the Instrument Section original articles describing new instruments or new methods for research or instruction in any branch of science such as Physics, Chemistry, Astronomy or Biology. The prize will be awarded by a committee to be appointed by the National Research Council.

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